Answer on Question #79082 – Math – Trigonometry

$$\sin^2 a+\cos^2 a\cdot\cos 2b=\cos^2 b-\sin^2 b\cdot\cos 2a \quad (1)$$

We shall use the equality

$$\cos 2x=\cos^2 x-\sin^2 x.$$

Then:

$$\sin^2 a+\cos^2 a\cdot(\cos^2 b-\sin^2 b)=\cos^2 b-\sin^2 b\cdot(\cos^2 a-\sin^2 a),$$
$$\sin^2 a+\cos^2 a\cdot\cos^2 b\underline{-\cos^2 a\cdot\sin^2 b}=\cos^2 b\underline{-\sin^2 b\cdot\cos^2 a}+\sin^2 b\cdot\sin^2 a,$$
$$\sin^2 a+\cos^2 a\cdot\cos^2 b=\cos^2 b+\sin^2 b\cdot\sin^2 a,$$
$$\sin^2 a-\sin^2 b\cdot\sin^2 a=\cos^2 b-\cos^2 a\cdot\cos^2 b,$$
$$\sin^2 a\cdot(1-\sin^2 b)=\cos^2 b\cdot(1-\cos^2 a),$$
$$\sin^2 a\cdot\cos^2 b=\cos^2 b\cdot\sin^2 a.$$

So,

$$\sin^2 a\cdot\cos^2 b=\sin^2 a\cdot\cos^2 b,$$

which is true.
If backward steps will be performed, then the initial formula (1) will be obtained.
It shows that the formula (1) is true.