

Answer on Question #79067 – Math – Calculus

Question

Integrate $\frac{x^2+1}{(2x+1)(x-1)(x+1)} dx$

Solution

Use the method of decomposition into simple ones. Decompose the integrand to the simplest terms:

$$\frac{x^2 + 1}{(2x + 1)(x - 1)(x + 1)} = \frac{A}{2x + 1} + \frac{B}{x - 1} + \frac{C}{x + 1}$$
$$\frac{x^2 + 1}{(2x + 1)(x - 1)(x + 1)} = \frac{A(x - 1)(x + 1) + B(2x + 1)(x + 1) + C(2x + 1)(x - 1)}{(2x + 1)(x - 1)(x + 1)}$$

The numerators of fractions with the same denominators are equal. Hence,

$$x^2 + 1 = A(x - 1)(x + 1) + B(2x + 1)(x + 1) + C(2x + 1)(x - 1)$$

Take into account that the coefficients for the same powers of x that are on the left and on the right must coincide.

$$x^2: 1 = A + 2B + 2C$$

$$x: 0 = 3B - C$$

$$x^0: 1 = -A + B - C$$

Solve the system.

$$C = 3B$$

$$\begin{cases} A + 2B + 6B = 1 \\ -A + B - 3B = 1 \end{cases}$$

$$8B - 2B = 2$$

$$6B = 2$$

$$B = \frac{1}{3}, A = -\frac{5}{3}, C = 1$$

Then

$$\begin{aligned}
\int \frac{x^2 + 1}{(2x + 1)(x - 1)(x + 1)} dx &= -\frac{5}{3} \int \frac{dx}{2x + 1} + \frac{1}{3} \int \frac{dx}{x - 1} + \int \frac{dx}{x + 1} = \\
&= -\frac{5}{3} \\
& * \frac{1}{2} \int \frac{d(2x + 1)}{2x + 1} + \frac{1}{3} \int \frac{d(x - 1)}{x - 1} + \int \frac{d(x + 1)}{x + 1} = -\frac{5}{6} \ln(2x + 1) \\
& + \frac{1}{3} \ln(x - 1) + \ln(x + 1) + C
\end{aligned}$$

Answer: $-\frac{5}{6} \ln(2x + 1) + \frac{1}{3} \ln(x - 1) + \ln(x + 1) + C.$