Answer on Question #79066 – Math – Calculus

Question

Integrate $\frac{1}{x^2-6x+11}dx$ within limit -5 to 5.

Solution

$$\int_{-5}^{5}\frac{1}{x^2-6x+11}dx$$

Indefinite integral

$$\int\frac{1}{x^2-6x+11}dx$$

$x^2-6x+11=x^2-6x+9+2=2+(x-3)^2$

Substitution

$u=\frac{x-3}{\sqrt{2}}$, $du=\frac{1}{\sqrt{2}}dx$

$2+(x-3)^2=2+2u^2=2(1+u^2)$

$$\int\frac{1}{x^2-6x+11}dx=\int\frac{\sqrt{2}}{2(1+u^2)}du=\frac{1}{\sqrt{2}}\arctan u+C=$$

$$=\frac{1}{\sqrt{2}}\arctan\left(\frac{x-3}{\sqrt{2}}\right)+C$$

$$\int_{-5}^{5}\frac{1}{x^2-6x+11}dx=\left[\frac{1}{\sqrt{2}}\arctan\left(\frac{x-3}{\sqrt{2}}\right)\right]_{-5}^{5}=$$

$$=\frac{1}{\sqrt{2}}\arctan\left(\frac{5-3}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\arctan\left(\frac{-5-3}{\sqrt{2}}\right)=$$

$$=\frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\right)+\frac{1}{\sqrt{2}}\arctan\left(4\sqrt{2}\right)\approx 1.6625097947$$

Answer: $\frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\right)+\frac{1}{\sqrt{2}}\arctan\left(4\sqrt{2}\right)\approx 1.6625097947$.