Answer on Question #79065 – Math – Calculus

Question

Find the maxima and minima, if any, for the function f given by

f(x)= x^4-14x^2+24x+6

Also find the values of the function at those points.

Solution

In order to find the extremums of a function (here we consider the case of functions with single variable), one should calculate its first derivative

$$f'(x) = 4x^3 - 28x + 24 \quad (1)$$

and find the roots of the equation

$$f'(x) = 0, \quad (2)$$

The simplest way of solving (2) with expression (1) consists in "guessing" the value of one of its roots (let it be x_1) and lowering the power of the initial polynomial by dividing it on the value $(x - x_1)$. It is easy to check that $x_1 = 1$. As a result, expression (1) can be factorized as

$$f'(x) = 4(x-1)(x^2 + x - 6), \quad (3)$$

It is quite simple to factorize the remaining part by obtaining the roots of the quadratic equation:

$$x_{2,3} = \frac{-1 \pm \sqrt{1+4\cdot 6}}{2}, \quad x_2 = 2, \quad x_3 = -3,$$

$$f'(x) = 4(x-1)(x-2)(x+3). \quad (4)$$

In order to distinguish between local minimums and maximums of the function, one should calculate the second derivative of the function and check its sign at points x_1, x_2 and x_3. The rule states that maximums are located at points where $f''(x_i) < 0$ and minimums - where $f''(x_i) > 0$.

$$f''(x) = 12x^2 - 28,$$

$$f''(1) = -16 < 0,$$

$$f''(2) = 20 > 0,$$

$$f''(-3) = 80 > 0 \, . \qquad (5)$$

Hence, the points $x_2 = 2$ and $x_3 = -3$ correspond to local minimums whereas the point $x_1 = 1$ gives a local maximum.

Finally, substituting these values into the initial function, we obtain:

(1, 17) is a local maximum; (2, 14) and (-3, -111) are local minimums.

Answer: (1, 17) is a local maximum; (2, 14) and (-3, -111) are local minimums.