

Answer on Question #79004 - Math - Trigonometry

$$\begin{aligned} a \cos^2 \theta + b \sin^2 \theta &= a(\cos^2 \theta + \sin^2 \theta) + (b - a) \sin^2 \theta = a + (b - a) \sin^2 \theta, \\ \Rightarrow a + (b - a) \sin^2 \theta &= c \end{aligned}$$

i.e.

$$\sin^2 \theta = \frac{c - a}{b - a} \quad (1)$$

$$\begin{aligned} a \cos^2 \theta + b \sin^2 \theta &= (a - b) \cos^2 \theta + b(\cos^2 \theta + \sin^2 \theta) = b + (a - b) \cos^2 \theta, \\ \Rightarrow b + (a - b) \cos^2 \theta &= c \end{aligned}$$

i.e.

$$\cos^2 \theta = \frac{c - b}{a - b} \quad (2)$$

Therefore

$$\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{a - c}{c - b} \quad (3)$$