Answer on Question #78913 – Math – Statistics and Probability

Question

A group of 25 students took examinations in both pure mathematics and statistics. Their marks out of 150 in mathematics, x, and in statistics, y, were recorded and are summarized below.

Σ x = 1978, Σ x2 = 175840 Σ y = 2123 Σ y2 = 202257 Σ xy = 181572

I. Calculate Sxx, Syy and Sxy.

II. Find the product moment correlation coefficient between the marks in pure Mathematics and Statistics.

III. Starting your hypotheses clearly tests, at the 5% level of significance, whether or not there is evidence of a correlation.

IV. State an assumption needed for the test in part (iii) to be valid.

Solution

We have:

$$\sum x = 1978, \sum x^2 = 175840$$

$$\sum y = 2123, \sum y^2 = 202257$$

$$\sum xy = 181572$$

I. $S_{xx} = \sum(x - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 175840 - \frac{1978^2}{25} = 19340.64$

$$S_{yy} = \sum(y - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 202257 - \frac{2123^2}{25} = 21971.84$$

$$S_{xy} = \sum(x - \bar{x})(y - \bar{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n} = 181572 - \frac{1978 * 2123}{25} = 13600.24$$

II. $r_{xy} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{13600.24}{\sqrt{19340.64 * 21971.84}} = 0.6597$

III. $H_0: p = 0$

$H_1: p \neq 0$, where p is the population correlation coefficient.

IV. Calculate the value of the test statistic using the following formula:

$$t^* = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$t^* = \frac{0.6597 * 23}{0.7515} = 20.19$$

$t_{0.95,23} = 2.06865$

Since $t^* > t_{0.95,23}$, we can reject H_0. There is sufficient statistical evidence at the α = 0.05 level to conclude that there is a significant linear relationship between x and y.

Answer:

I. $S_{xx} = 19340.64$

$S_{yy} = 21971.84$

$S_{xy} = 13600.24$

II. $r_{xy} = 0.6597$

III. $H_0: p = 0$

$H_1: p \neq 0,$

IV. There is sufficient statistical evidence at the α = 0.05 level to conclude that there is a significant linear relationship between x and y.