ANSWER on Question #78911 – Math – Linear Algebra

QUESTION

A firm uses two inputs, K and L to manufacture final product. The price per unit of these inputs are $sh.20$ and $sh.4$ respectively. What combination of inputs should a firm use to maximize output given that the budget is fixed at $sh.390$?

SOLUTION

Let the manufactory of K produce x units of the finished product, and the manufactory of L - y units.

Then, the total number of products produced is

$$Units(x,y) = x + y$$

According to the problem: the manufacture K pay $sh.20$ for each unit and the manufacture L - $sh.4$.

Then, the total amount of money spent is

$$Cost(x,y) = 20x + 4y$$

By the condition of the task, the amount of money spent is fixed and equal

$$Cost(x,y) = 390 \to \boxed{20x + 4y = 390}$$

Now we can write the given problem in symbolic form:

$$\begin{cases} \max\left(Cost(x+y)\right) = \max(x+y) \\ 20x + 4y = 390 \\ x \geq 0 \\ y \geq 0 \\ x, y \in \mathbb{N} \end{cases}$$

Then,

$$20x + 4y = 390 \to 4y = 390 - 20x| \div (4) \to y = \frac{390}{4} - \frac{20x}{4} \to \boxed{y = \frac{195}{2} - 5x}$$

$$\max(x,y) = x + y = \frac{195}{2} - 5x + x = \frac{195}{2} - 4x \to \boxed{\max(x,y) = \frac{195}{2} - 4x}$$

As we can see,

$$\max(x,y)=\frac{195}{2}-4x-\ decreasing\ linear\ function\ \forall x\geq 0$$

Then,

$$\max(x,y)=\frac{195}{2}-4x\rightarrow\frac{195}{2}-for\ x=0.$$

$$\begin{cases}20x+4y=390\\ x=0\end{cases}\rightarrow 4y=390\rightarrow y=\frac{195}{2}=97.5$$

But, y is the number of products manufactured, it can only be an integer, so $\boxed{y=97}$.

Conclusion,

$$\max\big(Cost(x,y)\big)=\max(x+y)=97$$

ANSWER:

$$\max_{\begin{cases}20x+4y=390\\ x,y\geq 0\end{cases}}(x+y)=97$$