Find the distance of the centre of the circle $x^2+y^2+z^2+x-2y+2z=3, \quad 2x+y+2z=1$ from the plane $ax+by+cz=d$ where a,b,c,d are constants and also find the equation of the right circular cylinder whose base curve is the circle obtained above.

Solution:

1.

$$x^2+y^2+z^2+x-2y+2z=3$$
$$x^2+2\cdot 0.5\cdot x+0.25+y^2-2y+1+z^2+2z+1-2.25=3$$
$$(x+0.5)^2+(y-1)^2+(z+1)^2=\frac{21}{4}$$

$x^2+y^2+z^2+x-2y+2z=3$ is an equation of the sphere with centre in $r_0=(-0.5,1,-1)$ and $R=\frac{\sqrt{21}}{2}$.

2. For plane $2x+y+2z=1$ normal vector is $v=(2,1,2)$.

3. Centre of the circle $x^2+y^2+z^2+x-2y+2z=3, \quad 2x+y+2z=1$ lies on radius perpendicular to the plane and can be found from following system:

$$\begin{cases} x_0=-0.5+2\cdot\lambda \\ y_0=1+1\cdot\lambda \\ z_0=-1+2\cdot\lambda \\ 2x_0+y_0+2z_0=1 \end{cases}$$
$$\begin{cases} \lambda=\frac{1}{3} \\ x_0=\frac{1}{6} \\ y_0=\frac{8}{6} \\ z_0=-\frac{2}{6} \end{cases}$$

Distance between centre of sphere and centre of circle: $h=\lambda\cdot\sqrt{2^2+1^2+2^2}=1$

4. Distance of the centre of the circle from the plane $ax+by+cz=d$ can be found from following expression:

$$d=\frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}=\frac{|a+8b-2c-6d|}{6\sqrt{a^2+b^2+c^2}}$$

5. Radius of circle: $R_c=\sqrt{R^2-h^2}=\sqrt{\frac{21}{4}-1}=\frac{\sqrt{17}}{2}$

6. Two orthonormal vectors on plane $2x+y+2z=1$: $\quad k=(-\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}), \qquad l=(\frac{\sqrt{2}}{6},-\frac{2\sqrt{2}}{3},\frac{\sqrt{2}}{6})$

7. Equation of right cylinder:

$$r(\varphi,\lambda) = r_0 + R_c(cos\varphi \cdot k + sin\varphi \cdot l) + \lambda \cdot v \qquad \varphi \in [0; 2\pi) \quad \lambda \in (-\infty; \infty)$$

with

$$r_0 = (-0.5, 1, -1)$$
$$R_c = \frac{\sqrt{17}}{2}$$
$$k = (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}), \qquad l = (\frac{\sqrt{2}}{6}, -\frac{2\sqrt{2}}{3}, \frac{\sqrt{2}}{6})$$
$$v = (2, 1, 2)$$

Answer:

Distance of the centre of the circle from the plane $d = \frac{|a+8b-2c-6d|}{6\sqrt{a^2+b^2+c^2}}$

Equation of cylinder: $r(\varphi,\lambda) = r_0 + R_c(cos\varphi \cdot k + sin\varphi \cdot l) + \lambda \cdot v$ (with the previously mentioned variables)