Answer on Question #78890 – Math – Analytic Geometry

Question

A rotating liquid forms a surface in the form of a paraboloid. The surface is 2m deep at the centre and 10m across. Obtain an equation of the surface.

Solution

The paraboloid which has radius a at height h is then given parametrically by

$x(u,\theta) = a\sqrt{u/h}\cos\theta$

$y(u,\theta) = a\sqrt{u/h}\sin\theta$

$z(u,\theta) = u$

$$x^2 + y^2 = a^2\left(\frac{u}{h}\right)\cos^2\theta + a^2\left(\frac{u}{h}\right)\sin^2\theta = a^2\left(\frac{u}{h}\right)$$

The equation of the surface

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = \frac{z}{h}$$

$$\frac{x^2}{2^2} + \frac{y^2}{2^2} = \frac{z}{10}$$

$$\frac{x^2}{4} + \frac{y^2}{4} = \frac{z}{10}$$

Answer: $\frac{x^2}{4} + \frac{y^2}{4} = \frac{z}{10}$.