Answer on Question #78863 - Math - Trigonometry

Question.

If $sin\,x + sin\,y = \sqrt{3}(cos\,y - cos\,x)$

Find value of $sin\,3x + sin\,3y$

Solution.

$$sin\,x + sin\,y = \sqrt{3}(cos\,y - cos\,x)$$

$$\frac{1}{2}sin\,x + \frac{1}{2}sin\,y = \frac{\sqrt{3}}{2}cos\,y - \frac{\sqrt{3}}{2}cos\,x$$

$$\frac{1}{2}sin\,x + \frac{\sqrt{3}}{2}cos\,x = \frac{\sqrt{3}}{2}cos\,y - \frac{1}{2}sin\,y$$

$$\frac{1}{2} = cos\,\frac{\pi}{3} \quad \frac{\sqrt{3}}{2} = sin\,\frac{\pi}{3}$$

$$cos\,\frac{\pi}{3}sin\,x + sin\,\frac{\pi}{3}cos\,x = sin\,\frac{\pi}{3}cos\,y - cos\,\frac{\pi}{3}sin\,y$$

$$sin\,(\frac{\pi}{3} + x) = sin\,(\frac{\pi}{3} - y)$$

$$sin\,(\frac{\pi}{3} + x) - sin\,(\frac{\pi}{3} - y) = 0$$

$$2sin\,(\frac{x+y}{2})cos\,(\frac{\pi}{3} + \frac{x-y}{2}) = 0 \;\Rightarrow \left(sin\,(\frac{x+y}{2}) = 0 \;\vee\; cos\,(\frac{\pi}{3} + \frac{x-y}{2}) = 0 \right)$$

$$1)\; sin\,(\frac{x+y}{2}) = 0 \Leftrightarrow x + y = 2n\pi,\; n \in \mathbb{Z} \Leftrightarrow x = -y + 2n\pi,\; n \in \mathbb{Z}$$

$$sin\,3x + sin\,3y = sin\,3(-y+2n\pi) + sin\,3y = sin\,(-3y+6n\pi) + sin\,3y = -sin\,3y + sin\,3y = 0,\; n \in \mathbb{Z}$$

$$2)\; cos\,(\frac{\pi}{3} + \frac{x-y}{2}) = 0 \Leftrightarrow x - y + \frac{2\pi}{3} = \pi + 2n\pi,\; n \in \mathbb{Z} \Leftrightarrow x = y + \frac{\pi}{3} + 2n\pi,\; n \in \mathbb{Z}$$

$$sin\,3x + sin\,3y = sin\,3(y + \frac{\pi}{3} + 2n\pi) + sin\,3y = sin\,(3y+\pi) + sin\,3y = -sin\,3y + sin\,3y = 0,\; n \in \mathbb{Z}$$

Answer: $sin\,3x + sin\,3y = 0$