

Answer on Question #78762 – Math – Analytic Geometry

Question

$ax^2+by^2+cz^2=d$ represents a sphere with radius $\sqrt{a^2+b^2+c^2-d}$ where a,b,c,d are positive real numbers. Is the statement true? Give reason for your answers either with a short proof or a counterexample.

Solution

If we divide the right and left sides of equation by d it will become

$$\frac{a}{d}x^2 + \frac{b}{d}y^2 + \frac{c}{d}z^2 = 1;$$

$$\frac{x^2}{m} + \frac{y^2}{n} + \frac{z^2}{p} = 1$$

We can see the canonical equation of ellipsoid, where m, n, p are also real positive numbers, they are called semiaxis of ellipsoid. Sphere is a particular case of an ellipsoid, when $m = n = p$. But it wasn't stated that $a = b = c$, so we can't say that $\frac{a}{d} = \frac{b}{d} = \frac{c}{d}$, so the statement is incorrect. But if $a = b = c$ the radius of the sphere will be $\sqrt{\frac{d}{a}}$, so in any way the statement is incorrect.

Answer: statement is incorrect.