Answer on Question #78657 – Math – Analytic Geometry

Question

Obtain the equation of the parabola with focus (3,2) and directrix 3x − 4y + 9 = 0.

Solution

Parabolas are commonly known as the graphs of quadratic functions. They can also be viewed as the set of all points whose distance from a certain point (the focus) is equal to their distance from a certain line (the directrix).

Given the focus and the directrix of a parabola, we can find the parabola's equation. Using the distance formula, we find that the distance between the point (x, y) on parabola and the point (x_F, y_F) of focus is

$$d_F = \sqrt{(x - x_F)^2 + (y - y_F)^2}.$$

On the other hand, the distance between the point (x, y) on parabola and the directrix $ax + by + c = 0$ is (as the distance between point and line)

$$d_D = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}.$$

So for parabola we have the equation

$$d_F = d_D; \ \ or \ \ d_F^2 = d_D^2;$$

$$(x - x_F)^2 + (y - y_F)^2 = \frac{(ax + by + c)^2}{a^2 + b^2}.$$

Solving this equation, we can get the parabola. In our case

$$(x_F, y_F) = (3,2);$$

$$a = 3; \ \ b = -4; \ \ c = 9.$$

Therefore, we have

$$(x - 3)^2 + (y - 2)^2 = \frac{(3\mathrm{x} \ - \ 4\mathrm{y} \ + \ 9)^2}{3^2 + 4^2};$$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = \frac{9x^2 + 16y^2 + 81 - 12xy + 27x - 12xy - 36y + 27x - 36y}{25};$$

$$25x^2 + 25y^2 - 150x - 100y + 325 = 9x^2 + 16y^2 - 24xy + 54x - 72y + 81;$$

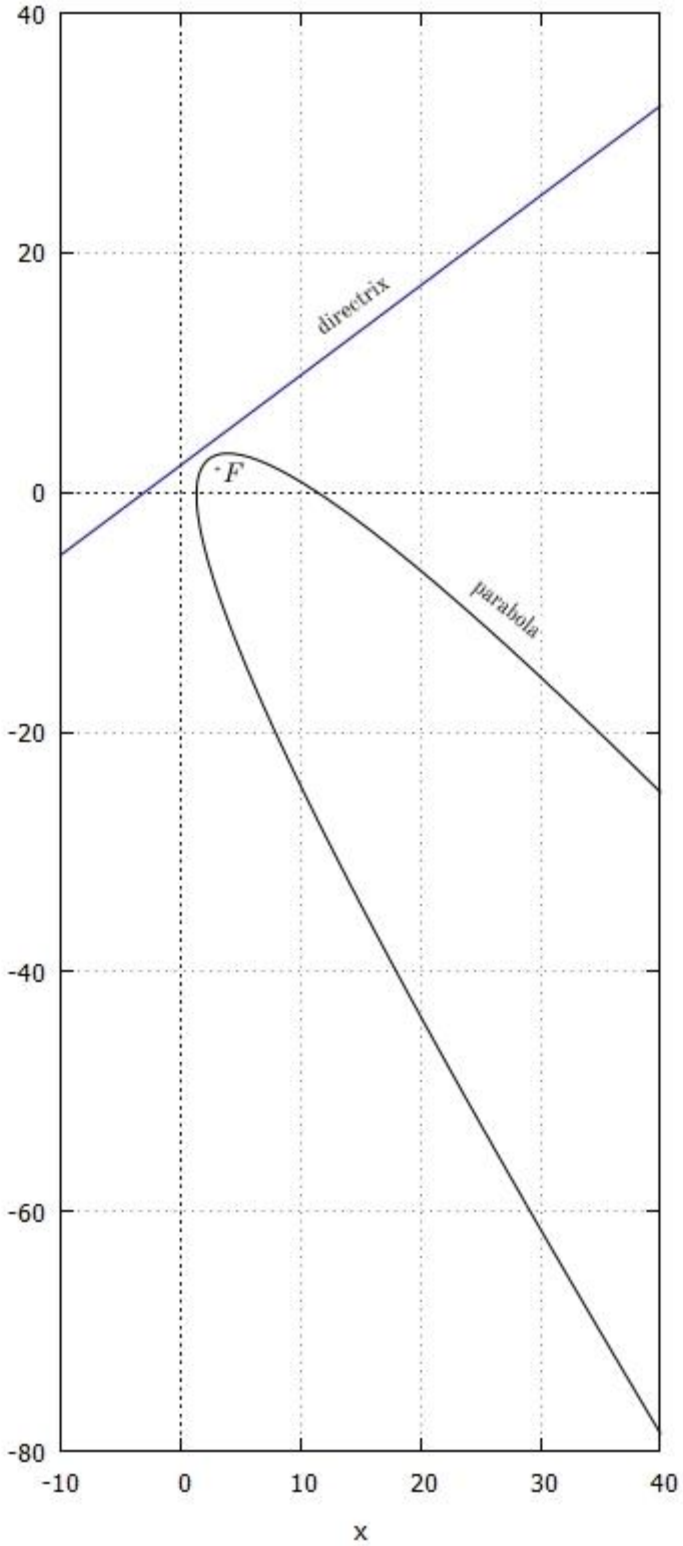
$$16x^2 + 24xy + 9y^2 - 204x - 28y + 244 = 0.$$

We get the implicit equation of a parabola defined by an irreducible polynomial of degree two.

Answer: the equation of the parabola is

$$16x^2 + 24xy + 9y^2 - 204x - 28y + 244 = 0.$$

The graph of the directrix, focus and our parabola is shown below



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20
0
-20
-40
-60
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40
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directrix
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