Answer on Question #78623 – Math – Analytic Geometry

Question

If $x/1 = y/1 = z/-1$ represents one of the three mutually perpendicular generators of the cone $3xy + 8xz - 5yz = 0$, find the equations of the other two.

Solution

Cone:

$$C \to ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

One of its generators:

$$L_1 \to \frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

Then L_1 must satisfy

$$ax^2 + bm^2 + cn^2 + 2fmn + 2gnl + 2hlm = 0$$

Now the plane $\Pi \to \langle p - p_0, \vec{v} \rangle$ with

$$p_0 = (0,0,0)$$

$$p = (x, y, z)$$

$$\vec{v} = (l, m, n)$$

is orthogonal to L_1

This plane cuts C in two other lines (L_2, L_3) such that $L_2 \perp L_3$ if

$$(a + b + c)(l^2 + m^2 + n^2) - C(l, m, n) = 0$$

or

$$(a + b + c)(l^2 + m^2 + n^2) = 0$$

or

$$a + b + c = 0$$

because $l^2 + m^2 + n^2 \neq 0$

So we have

$$\vec{v} = (l, m, n) = (1,1,-1)$$

$$f = -5, g = 8, h = 3$$

Then solving

$$\begin{cases} fyz + gzx + hxy = 0 \\ lx + my + nz = 0 \end{cases}$$

we obtain L_2, L_3 as follows

$$L_2 = \begin{cases} x = \dfrac{z}{3} \\ y = \dfrac{2}{3}z \end{cases}$$

$$L_3 = \begin{cases} x = 5z \\ y = -4z \end{cases}$$