

Answer on Question #78515 – Math – Analytic Geometry

Question

$x + y + z = 0$ touches the cone $x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$

Is the statement true? Give reason for your answer, either with a short proof or a counterexample.

Solution

$$\begin{cases} x + y + z = 0 \\ x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0 \end{cases}$$

If the system has only one solution, then plane touches the cone.

We have 3 variables and 2 equations. The system has more than one solution, for example,

$$(x, y, z) = (0, 0, 0) \text{ and } (x, y, z) = (-1, 1, 0).$$

Thus, $x + y + z = 0$ does not touch, it crosses the cone $x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$.