Answer on Question #78507 - Math - Discrete Mathematics

June 26, 2018

Question. For any two sets A and B, in a universal set U, prove that

$$A \subseteq B \iff A \cup B = B.$$

Answer. Assume that A and B are sets in a universal set U.

First, we will prove the implication from left to right. Assume that $A \subseteq B$. We need to prove $A \cup B = B$ which is equivalent to

$$x \in A \cup B \iff x \in B$$

for every $x \in U$.

- (From left to right.) Let $x \in A \cup B$. Then $x \in A$ or $x \in B$ by the definition of union.

  - Assume $x \in A$. Then $x \in B$ by the assumption $A \subseteq B$.
  - Assume $x \in B$. Then $x \in B$.

  Hence in both cases $x \in B$.

- (From right to left.) If $x \in B$, then $x \in A \cup B$ by the definition of union.

Second, we will prove the implication from right to left. Assume that $A \cup B = B$. We need to prove $A \subseteq B$. Let $x \in A$. Then $x \in A \cup B$ by the definition of union. From the assumption $A \cup B = B$, it follows that $x \in A \cup B$ implies $x \in B$ for every $x \in U$. Hence $x \in B$.