Question

Find the roots of the equation $2x^3 - x^2 - 22x - 24 = 0$ if two of them are in the ratio 4:3.

Solution

Let's denote roots as x_1, $x_2 = 4a$ and $x_3 = 3a$, where roots x_2 and x_3 satisfy the given relation. So we only have to find x_1 and a.

If the roots of the given equation are known then it can be written as follows

$$2(x - x_1)(x - x_2)(x - x_3) = 0$$

Substituting $x_2 = 4a$ and $x_3 = 3a$ and expanding we obtain

$$2x^3 - (2x_1 + 14a)x^2 + (14ax_1 + 24a^2)x - 24a^2x_1 = 0$$

Equating the coefficients for corresponding degrees of x of this equation and the given one, we obtain

$$2 = 2$$

$$2x_1 + 14a = 1$$

$$14ax_1 + 24a^2 = -22$$

$$24a^2x_1 = 24$$

From the second equation:

$$2x_1 = 1 - 14a$$

Substituting into the third equation we obtain

$$7a(1 - 14a) + 24a^2 = -22$$

$$74a^2 - 7a - 22 = 0$$

$$a = -\frac{1}{2}, \frac{22}{37}$$

Lets take $a = -1/2$. Since $a^2x_1 = 1$ (4th equation), we obtain

$$x_1 = \frac{1}{a^2} = 4, x_2 = 4a = -2, x_3 = 3a = -\frac{3}{2}$$

By substitution we can verify that these are indeed the roots of the given equation.

Answer: $4, -2, -3/2$.