Answer on Question #78501 – Math – Other

Question

Find the polynomial over R of least degree which has $i - 3$ and $\sqrt{7} + 5i$ as its roots.

Solution

The polynomial of least degree, which has roots $x_1 = i - 3$ and $x_2 = \sqrt{7} + 5i$ is given by

$$P(x) = (x - x_1)(x - x_2)$$

But it has complex coefficients. To get the polynomial $Q(x)$ of least degree over $\mathbb{R}$ we must multiply polynomial $P(x)$ by $(x - \overline{x_1})(x - \overline{x_2})$, where $\overline{x_1} = -i - 3$, $\overline{x_2} = \sqrt{7} - 5i$. Thus we obtain

$$Q(x) = (x - x_1)(x - \overline{x_1})(x - x_2)(x - \overline{x_2}) =$$

$$= (x^2 - x(x_1 + \overline{x_1}) + \overline{x_1}x_1)(x^2 - x(x_2 + \overline{x_2}) + \overline{x_2}x_2) =$$

$$= (x^2 + 6x + 10)\left(x^2 - 2\sqrt{7}x + 32\right) =$$

$$= 320 + 192x - 20\sqrt{7}x + 42x^2 - 12\sqrt{7}x^2 + 6x^3 - 2\sqrt{7}x^3 + x^4$$

Answer: $320 + 192x - 20\sqrt{7}x + 42x^2 - 12\sqrt{7}x^2 + 6x^3 - 2\sqrt{7}x^3 + x^4$.