

Answer on Question #78495 – Math – Linear Algebra

Question

Apply the Gaussian elimination process to determine values of λ for which the following linear system is consistent:

$$x - 3y + 4 = 0$$

$$3x - 2y = \lambda$$

$$y = 6 - 2x$$

Solution

$$x - 3y + 4 = 0$$

$$3x - 2y = \lambda$$

$$y = 6 - 2x$$

$$x - 3y = -4$$

$$3x - 2y = \lambda$$

$$2x + y = 6$$

Augmented matrix

$$\begin{pmatrix} 1 & -3 & -4 \\ 3 & -2 & \lambda \\ 2 & 1 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & -4 \\ 3 & -2 & \lambda \\ 2 & 1 & 6 \end{pmatrix} \xrightarrow{R_2 - (3)R_1} \begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & \lambda + 12 \\ 2 & 1 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & \lambda + 12 \\ 2 & 1 & 6 \end{pmatrix} \xrightarrow{R_3 - (2)R_1} \begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & \lambda + 12 \\ 0 & 7 & 14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & \lambda + 12 \\ 0 & 7 & 14 \end{pmatrix} \xrightarrow{R_3 - R_2} \begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & \lambda + 12 \\ 0 & 0 & 2 - \lambda \end{pmatrix}$$

The following linear system is consistent if

$$2 - \lambda = 0 \Rightarrow \lambda = 2$$

Then

$$\begin{pmatrix} 1 & -3 & -4 \\ 0 & 7 & 14 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_2/7} \begin{pmatrix} 1 & -3 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 + (3)R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

The solution is $x = 2, y = 2$.

Answer: the system is consistent if $\lambda = 2$, then $x = y = 2$.