Answer on Question #78477 – Math – Linear Algebra

Question

Give examples, with justification, of the following:

two non-singular 2×2 matrices C and D, with |C| =√2 |D|

Solution

Since we have to satisfy only $|C| = \sqrt{2}\,|D|$, we can choose matrix D arbitrary. Let it be equal to unity matrix E_2:

$$D = E_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then

$$|D| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

To satisfy the given condition we can create matrix C from matrix D by changing upper left element to $\sqrt{2}$:

$$C = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

Indeed,

$$|C| = \begin{vmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{vmatrix} = \sqrt{2} \cdot 1 = \sqrt{2} = |D|$$

Answer: $C = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.