Answer on Question #78475 – Math – Linear Algebra

Question

Is Cramer's Rule applicable for solving the linear system below? If yes, apply it. Otherwise, alter the last equation in the system so that the solution can be obtained by applying the Rule.

$$x + y + z = \pi$$

$$-\pi x + \pi y + \sqrt{2} z = 0$$

$$\pi^2 x + \pi^2 y + 2z = 0$$

Solution

The given equation in matrix form

$$\begin{pmatrix} 1 & 1 & 1 \\ -\pi & \pi & \sqrt{2} \\ \pi^2 & \pi^2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \pi \\ 0 \\ 0 \end{pmatrix}$$

This equation can be solved by Cramer's Rule, if the determinant of the matrix doesn't vanish.

$$\begin{vmatrix} 1 & 1 & 1 \\ -\pi & \pi & \sqrt{2} \\ \pi^2 & \pi^2 & 2 \end{vmatrix} = \begin{vmatrix} \pi & \sqrt{2} \\ \pi^2 & 2 \end{vmatrix} - \begin{vmatrix} -\pi & \sqrt{2} \\ \pi^2 & 2 \end{vmatrix} + \begin{vmatrix} -\pi & \pi \\ \pi^2 & \pi^2 \end{vmatrix} =$$

$$= 2\pi - \sqrt{2}\pi^2 - \left(-2\pi - \sqrt{2}\pi^2\right) - \pi^3 - \pi^3 = 4\pi - 2\pi^3 \neq 0$$

Thus the Cramer's Rule is applicable. Then the solution is given by

$$x = \frac{\begin{vmatrix} \pi & 1 & 1 \\ 0 & \pi & \sqrt{2} \\ 0 & \pi^2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ -\pi & \pi & \sqrt{2} \\ \pi^2 & \pi^2 & 2 \end{vmatrix}} = \frac{\pi\left(2\pi - \sqrt{2}\pi^2\right)}{4\pi - 2\pi^3} = \frac{\pi\sqrt{2}\left(\sqrt{2} - \pi\right)}{2(2 - \pi^2)} = \frac{\pi}{\sqrt{2}\left(\sqrt{2} + \pi\right)}$$

$$y = \frac{\begin{vmatrix} 1 & \pi & 1 \\ -\pi & 0 & \sqrt{2} \\ \pi^2 & 0 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ -\pi & \pi & \sqrt{2} \\ \pi^2 & \pi^2 & 2 \end{vmatrix}} = \frac{-\pi\left(-2\pi - \sqrt{2}\pi^2\right)}{4\pi - 2\pi^3} = \frac{\pi\left(2 + \sqrt{2}\pi\right)}{2(2 - \pi^2)} = \frac{\pi}{\sqrt{2}\left(\sqrt{2} - \pi\right)}$$

$$z=\frac{\begin{vmatrix}1 & 1 & \pi\\ -\pi & \pi & 0\\ \pi^2 & \pi^2 & 0\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ -\pi & \pi & \sqrt{2}\\ \pi^2 & \pi^2 & 2\end{vmatrix}}=\frac{\pi(-\pi^3-\pi^3)}{4\pi-2\pi^3}=\frac{\pi^3}{2-\pi^2}$$

Answer: $\left(\frac{\pi}{\sqrt{2}(\sqrt{2}+\pi)},\frac{\pi}{\sqrt{2}(\sqrt{2}-\pi)},\frac{\pi^3}{2-\pi^2}\right)$.