Answer on Question #78458 – Math – Algebra

Question

Show that

$$1 + 1/\sqrt{2} + \ldots + 1/\sqrt{n} \geq \sqrt{2}/(n-1) \; for \; n \; \in N, n \; > \; 1$$

Solution

Induction method:

$$for \; n = 2: \; 1 + 1/\sqrt{2} \geq \sqrt{2}$$
$$\sqrt{2} + 1 \geq 2$$
$$\sqrt{2} \geq 1,$$

which is true,

and if

$$1 + 1/\sqrt{2} + \ldots + 1/\sqrt{n} \geq \sqrt{2}/(n-1)$$

Then we have to prove that

$$1 + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} \geq \frac{\sqrt{2}}{n+1-1} = \frac{\sqrt{2}}{n}$$

Proof

Since

$$1 + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \geq \frac{\sqrt{2}}{n-1}$$

and

$$\frac{\sqrt{2}}{n-1} \geq \frac{\sqrt{2}}{n}$$

Then

$$1 + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} \geq \frac{\sqrt{2}}{n-1} \geq \frac{\sqrt{2}}{n}$$

which was to be proved.