Condition: Apply the Gaussian elimination process to determine the value of lembda for which the following linear system is consistent: x-3y+4=0, 3x-2y=lembda, y= 6-2x ?

Solution:

Firstly, let us transform the form of the given equations to normal form.

$$\begin{cases} x-3y=-4 \\ 3x-2y=\lambda \\ 2x+y=6 \end{cases}$$

Now, create augmented matrix of the given system.

$$\left(\begin{array}{cc|c} 1 & -3 & -4 \\ 3 & -2 & \lambda \\ 2 & 1 & 6 \end{array}\right)$$

Where the first column is coefficients of the variable x, the second is coefficients of y.

Then, using the Gaussian method of elimination, let us find our solution.

1) In the first step we compose the first linear equation and -3, then the result we add to the second linear equation. After we again compose the first linear equation and -2, then the result we add to the third linear equation.

The result of the first step is:

$$\left(\begin{array}{cc|c} 1 & -3 & -4 \\ 0 & 7 & 12+\lambda \\ 0 & 7 & 6 \end{array}\right)$$

2) In the second step we compose the third linear equation and -1, then we add result to the third second equation

The result of the second step is:

$$\left(\begin{array}{cc|c} 1 & -3 & -4 \\ 0 & 0 & 6+\lambda \\ 0 & 7 & 6 \end{array}\right)$$

3) Let us analyze the second equation. For doing that, rewrite it in the normal form of the equation:

$$0*x+0*y=6+\lambda$$

So, this equation will be right if λ = -6 and because of that linear system will be consistent.

If take different value of λ, then we will get this $0 + 0 = k$, where $k \neq 0$. This mean that there is no x or y that satisfy this equation. Because of that system will be inconsistent.

Answer: $\lambda = -6$;