

**Question 1.** Let  $x_1 = a > 0$  and  $x_{n+1} = x_n + 1/x_n$ . Show that  $x_n$  diverges.

*Solution.* Suppose there is  $b = \lim x_n < \infty$ . Note that  $b > 0$ , because  $x_1 > 0$  and  $(x_n)$  is increasing. Then consider the equality  $x_{n+1} = x_n + 1/x_n$ . As  $n \rightarrow \infty$  the left-hand side of this equality tends to  $b$ , while the right-hand side tends to  $b + 1/b$ . Since  $b \neq b + 1/b$ , we obtain a contradiction.  $\square$