

**Question 1.** Show that it is possible to have a sequence  $(x_n)$  diverge while the sequence  $(|x_n|)$  converges.

*Solution.* Consider  $x_n = (-1)^n$ . Then  $(x_n)$  diverges, because there are two subsequences  $x_{2n} = 1$  and  $x_{2n-1} = -1$ , which have different limits (1 and  $-1$ , respectively). But  $|x_n| = 1$  for all  $n$ , so  $(|x_n|)$  obviously converges to 1.  $\square$