Question 1. *(a) Give an example to show it is possible to have* $\lim(x_n+y_n)$ *exist without having* $\lim x_n$ *or* $\lim y_n$ *exist.*

(b) Give an example to show it is possible to have $\lim(x_n y_n)$ *exist without having* $\lim x_n$ *or* $\lim y_n$ *exist.*

Solution. (a) Consider $x_n = (-1)^n$, $y_n = -x_n$. Then $x_n + y_n = 0$ for all $n \in \mathbb{N}$, so $\lim(x_n + y_n) = 0$. But $\lim x_n$ does not exist, because there are two subsequences $x_{2n} = 1$ and $x_{2n-1} = -1$, which have different limits (1 and -1, respectively). Similarly y_n does not have a limit.

(b) Take $x_n = y_n = (-1)^n$. As was shown above, this sequence does not have a limit. Nevertheless $x_n y_n = (-1)^{2n} = 1$ for all n, therefore, $\lim(x_n y_n) = 1$. □