Answer on Question #78301 – Math – Linear Algebra

Question

1a) what would be the gradient of a line parallel to the straight line
$$3x - y + 4 = 0$$

Solution
$$3x - y + 4 = 0$$
The equation of the straight line in slope-intercept form
$$y = 3x + 4$$
The gradient of a line parallel to the given straight line is
$$grad = m = 3$$
Answer: 3.

Question

b) what would be the gradient of a line perpendicular to the straight line
$$5x - 2y - 1 = 0$$

Solution
$$5x - 2y - 1 = 0$$
The equation of the straight line in slope-intercept form
$$y = \frac{5}{2}x - \frac{1}{2}$$
If two lines are perpendicular
$$grad_1 \cdot grad_2 = -1$$
The gradient of a line perpendicular to the given straight line is
$$grad_2 = -\frac{1}{grad_1} = -\frac{1}{\frac{5}{2}} = -\frac{2}{5}$$

Answer: $-\frac{2}{5}$.

2. State whether the following pairs of lines whose equations are given are parallel, perpendicular or neither.

Question

a. $2x - y + 4 = 0$ and $6x - 3y + 7 = 0$

Solution
$2x - y + 4 = 0$ and $6x - 3y + 7 = 0$
The equations of the straight line in slope-intercept form
$$y = 2x + 4 \quad and \quad y = 2x + \frac{7}{3}$$
$$grad_1 = 2 = grad_2$$

Therefore, two lines are parallel.
Answer: two lines are parallel.

Question

b. $7x + 3y - 8 = 0$ and $3x - 7y + 1 = 0$

Solution

$7x + 3y - 8 = 0$ and $3x - 7y + 1 = 0$

The equations of the straight line in slope-intercept form

$$y = -\frac{7}{3}x + \frac{8}{3} \quad \text{and} \quad y = \frac{3}{7}x + \frac{1}{7}$$

$$grad_1 = -\frac{7}{3}, grad_2 = \frac{3}{7}$$

$$grad_1 \cdot grad_2 = -\frac{7}{3} \cdot \frac{3}{7} = -1$$

Therefore, two lines are perpendicular.
Answer: two lines are perpendicular.

Question

c. $x + 3y - 2 = 0$ and $3x - y + 4 = 0$

Solution

$x + 3y - 2 = 0$ and $3x - y + 4 = 0$

The equations of the straight line in slope-intercept form

$$y = -\frac{1}{3}x + \frac{2}{3} \quad \text{and} \quad y = 3x + 4$$

$$grad_1 = -\frac{1}{3}, grad_2 = 3$$

$$grad_1 \cdot grad_2 = -\frac{1}{3} \cdot 3 = -1$$

Therefore, two lines are perpendicular.
Answer: two lines are perpendicular.

3. Find the equation of the straight lines

Question

a) Passing through the point $(3, -2)$ and parallel to the line $4x - y + 6 = 0$

Solution

$$4x - y + 6 = 0$$

The equation of the straight line in slope-intercept form

$$y = 4x + 6$$

Two lines are parallel, then
$$grad_1 = grad_2 = 4$$

The equation of new straight line in slope-intercept form
$$y = grad_2 \cdot x + b_2$$
$$y = 4x + b_2$$
This line passes through the point $(3, -2)$
$$-2 = 4(3) + b_2$$
$$b_2 = -14$$
The equation of new straight line in slope-intercept form
$$y = 4x - 14$$
The equation of new straight line in general form
$$4x - y - 14 = 0$$
Answer: $4x - y - 14 = 0$.

Question

b) Passing through the origin and parallel to the line $5x + 3y - 7 = 0$

Solution
$$5x + 3y - 7 = 0$$
The equation of the straight line in slope-intercept form
$$y = -\frac{5}{3}x + \frac{7}{3}$$
Two lines are parallel, then
$$grad_1 = grad_2 = -\frac{5}{3}$$
The equation of a new straight line in slope-intercept form
$$y = grad_2 \cdot x + b_2$$
$$y = -\frac{5}{3}x + b_2$$
This line passes through the origin
$$0 = -\frac{5}{3}(0) + b_2$$
$$b_2 = 0$$
The equation of a new straight line in slope-intercept form
$$y = -\frac{5}{3}x$$
The equation of a new straight line in general form
$$5x + 3y = 0$$
Answer: $5x + 3y = 0$.

Question

c) Passing through the point $(-2,5)$ and perpendicular to the line

$$3x - 2y + 8 = 0$$

Solution
$$3x - 2y + 8 = 0$$
The equation of the straight line in slope-intercept form
$$y = \frac{3}{2}x + 4$$
Two lines are perpendicular, then
$$grad_1 \cdot grad_2 = -1$$
$$\frac{3}{2} \cdot grad_2 = -1$$
$$grad_2 = -\frac{2}{3}$$
The equation of new straight line in slope-intercept form
$$y = grad_2 \cdot x + b_2$$
$$y = -\frac{2}{3}x + b_2$$
This line passes through the point $(-2,5)$
$$5 = -\frac{2}{3}(-2) + b_2$$
$$b_2 = \frac{11}{3}$$
The equation of new straight line in slope-intercept form
$$y = -\frac{2}{3}x + \frac{11}{3}$$
The equation of new straight line in general form
$$2x + 3y - 11 = 0$$
Answer: $2x + 3y - 11 = 0$.