Answer on Question #78072 - Math - Calculus

A piece of wire 14 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle.

a) How much wire should be used for the square in order to maximize the total area?
b) How much wire should be used for the square in order to minimize the total area?

Solution:

Let x be the piece of wire from which the square was made, and y the piece of wire from which the circle was made. Then

$$x + y = 14, \qquad A_{sq} = \left(\frac{x}{4}\right)^2, \qquad A_c = \frac{y^2}{4\pi}.$$

where A_{sq} – area of a square, A_c – area of a circle.

$$A_{total} = A_{sq} + A_c = \frac{x^2}{16} + \frac{y^2}{4\pi} = \frac{x^2}{16} + \frac{(14-x)^2}{4\pi} = \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{7}{\pi}x + \frac{49}{\pi}.$$

It can be seen that the function $A_{total}(x)$ is a parabola with branches pointing upwards. The minimum of the function lies at the vertex of the parabalo, which is obvious. Since the branches of the parabola are directed upwards, the maximum value in the interval $0 \le x \le 14$ will be on the boundaries of this interval.

$$A_{total}(0) = \frac{49}{\pi} \approx 15.6\,, \qquad A_{total}(14) = \frac{49}{4} = 12.25\,.$$

We have $A_{total}(0) > A_{total}(14)$, hence the maximum of the function is on the value $x = 0$. For a parabola of the form $y = ax^2 + bx + c$, the coordinates of the vertex $O(m,n)$ are given by the formulas:

$$m = -\frac{b}{2a}, \qquad n = am^2 + bm + c.$$

$$m = -\frac{-\frac{7}{\pi}}{2\left(\frac{1}{16} + \frac{1}{4\pi}\right)} = \frac{56}{\pi + 4} \approx 7.84.$$

$$A_{total}(m) = n \approx 6.862\,.$$

Answer:

a) at x equal to 0 the total area is maximal;
b) at x equal to 7.84 the total area is minimal.