The position of an object moving along a line is given by the function $s(t) = 6t^2 + 36t$. Find the average velocity of the object over the following intervals.

a) [1, 7]
b) [1, 6]
c) [1, 5]
d) [1, 1 plus *h*] where *h* > 0 is real number.

Solution:

General equations for the position of the body and its velocity for uniformly accelerated motion have the form:

$$s(t) = v_0 t + \frac{at^2}{2},$$
$$v(t) = v_0 + at,$$

where v_0 – initial velocity and *a* – acceleration. From the condition of the problem it is seen that the initial velocity and acceleration are respectively equal to 36 and 12. Then

$$v(t) = 36 + 12t.$$

The average velocity in the interval [t_1, t_2] for uniformly accelerated motion is equal to the average value between the initial and final velocities for a given interval.

$$v_{average} = \frac{v(t_1) + v(t_2)}{2}.$$

or in this task

$$v_{average} = \frac{36 + 12t_1 + 36 + 12t_2}{2} = \frac{72 + 12(t_1 + t_2)}{2} = 36 + 6(t_1 + t_2).$$

a) $v_{average} = 36 + 6(1 + 7) = 84$;
b) $v_{average} = 36 + 6(1 + 6) = 78$;
c) $v_{average} = 36 + 6(1 + 5) = 72$;
d) $v_{average} = 36 + 6(1 + 1 + h) = 48 + 6h$.

Answer:

a) $v_{average} = 84$;
b) $v_{average} = 78$;
c) $v_{average} = 72$;
d) $v_{average} = 48 + 6h$.