

Answer on Question #78050 – Math – Calculus

Question

A force $\vec{F} = 3\vec{i} - 6\vec{k}$ acts along a line passing through the point $P(0, -1, 4)$. Determine the torque about the point $Q(4, 6, -1)$.

Solution

By definition, the torque is

$$\vec{\tau} = \vec{r} \times \vec{F}.$$

Here \vec{r} is the position vector of the point of application of force with respect to the point about which torque is to be calculated, \vec{F} is the force applied, $\vec{\tau}$ is the torque.

In our case, if Q is the center of rotation, and P is the point the force is applied, then $\vec{r} = \overrightarrow{QP}$. Then we have

$$\vec{r} = \overrightarrow{QP} = (0 - 4)\vec{i} + (-1 - 6)\vec{j} + (4 + 1)\vec{k} = -4\vec{i} - 7\vec{j} + 5\vec{k};$$

$$\vec{F} = 3\vec{i} + 0\vec{j} - 6\vec{k}.$$

Therefore, the torque is

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -7 & 5 \\ 3 & 0 & -6 \end{vmatrix} = \vec{i}(42 - 0) - \vec{j}(24 - 15) + \vec{k}(0 + 21) = 42\vec{i} - 9\vec{j} + 21\vec{k}.$$

Answer: $\vec{\tau} = 42\vec{i} - 9\vec{j} + 21\vec{k}.$