Answer on Question #77927 – Math – Analytic Geometry

Question

Center of the circle (0,0) is tangent to the line x-3y =6

Solution

If the line is tangent to the circle, they have only one common point, and system $\begin{cases} x^2 + y^2 = R^2 \\ x - 3y = 6 \end{cases}$, where $x^2 + y^2 = R^2$ is an equation of a circle with center at (0,0) and radius R, has only one real root (x_0, y_0).

$$\begin{cases} x = 3y + 6 \\ (3y+6)^2 + y^2 = R^2 \end{cases}; \begin{cases} x = 3y + 6 \\ 9y^2 + 36y + 36 + y^2 = R^2 \end{cases}; \begin{cases} x = 3y + 6 \\ 10y^2 + 36y + 36 - R^2 = 0 \end{cases}$$

And if the system has only one real root, equation $10y^2 + 36y + (36 - R^2) = 0$ must have only one real root too. Its discriminant is $D = b^2 - 4ac = 36^2 - 4 \times 10 \times (36 - R^2) = 1296 - 1440 + 40R^2$. But if square equation has only one real root – its discriminant is 0

$$1296 - 1440 + 40R^2 = 0;$$

$$40R^2 = 144$$

$$R^2 = 3.6$$

So, the equation of a circle is $x^2 + y^2 = 3.6$. From the equation $10y^2 + 36y + (36 - R^2) = 0$ we can find $y_0 = -\frac{b}{2a} = -\frac{36}{2 \times 10} = -\frac{36}{20} = -1.8$. And from the equation $x = 3y + 6$ we can find $x_0 = 3y_0 + +6 = 3 \times (-1.8) + 6 = -5.4 + 6 = 0.6$

Answer: Equation of a circle is $x^2 + y^2 = 3.6$, and point of contact is (0.6, -1.8).