

Answer on Question #77852 – Math – Calculus

Question

Calculate a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

Solution

$$f(x, y, z) = x^3 + y^3 + 3xyz - 3 = 0.$$

The gradient of $f(x, y, z)$ at point x, y, z is a vector normal to the surface at this point.

$$\nabla f(x, y, z) = (f_x, f_y, f_z) = (3x^2 + 3yz, 3y^2 + 3xz, 3xy).$$

At the point $(1, 2, -1)$: $\nabla f(x, y, z) = (-3, 9, 6)$.

$$\text{A unit vector normal: } \frac{1}{\sqrt{3^2+9^2+6^2}}(-3, 9, 6) = \left(-\frac{1}{\sqrt{14}}, \frac{13}{\sqrt{14}}, \sqrt{\frac{2}{7}}\right).$$

$$\text{Answer: } \left(-\frac{1}{\sqrt{14}}, \frac{13}{\sqrt{14}}, \sqrt{\frac{2}{7}}\right).$$