

## Answer on Question #77852 – Math – Calculus

### Question

Calculate a unit vector normal to the surface  $x^3 + y^3 + 3xyz = 3$  at the point  $(1, 2, -1)$ .

### Solution

$$f(x, y, z) = x^3 + y^3 + 3xyz - 3 = 0.$$

The gradient of  $f(x, y, z)$  at point  $x, y, z$  is a vector normal to the surface at this point.

$$\nabla f(x, y, z) = (f_x, f_y, f_z) = (3x^2 + 3yz, 3y^2 + 3xz, 3xy).$$

At the point  $(1, 2, -1)$ :  $\nabla f(x, y, z) = (-3, 9, 6)$ .

$$\text{A unit vector normal: } \frac{1}{\sqrt{3^2+9^2+6^2}}(-3, 9, 6) = \left(-\frac{1}{\sqrt{14}}, \frac{13}{\sqrt{14}}, \sqrt{\frac{2}{7}}\right).$$

$$\text{Answer: } \left(-\frac{1}{\sqrt{14}}, \frac{13}{\sqrt{14}}, \sqrt{\frac{2}{7}}\right).$$