ANSWER on Question #77851 – Math – Calculus

QUESTION

A rigid body is rotating with an angular speed of $\omega = 3 \ \ rad \cdot s^{-1}$ about an axis

$$\overrightarrow{OL} = 2\vec{\imath} - 2\vec{\jmath} + \vec{k}$$

where O is the origin. Determine the velocity of the body at the point $P(4,1,2)$.

SOLUTION

By the definition, the relationship between the angular velocity and the velocity has the form

$$\vec{v} = \vec{\omega} \times \vec{r} = \vec{v} = (\omega \cdot \vec{u}) \times \vec{r},$$

where

$$\vec{u} - unit\ vector\ of\ the\ direction\ of\ the\ axis\ of\ rotation$$

$$\vec{r} - radius - vector\ of\ the\ point\ at\ which\ we\ want\ to\ know\ the\ speed$$

$$\vec{\omega} \times \vec{r} - cross\ product$$

(More information: https://en.wikipedia.org/wiki/Angular_velocity)

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In our case,

$$p.P(4,1,2) \rightarrow \boxed{\vec{r} = 4\vec{\imath} + 1\vec{\jmath} + 2\vec{k} \leftrightarrow \begin{cases} r_x = 4 \\ r_y = 1 \\ r_z = 2 \end{cases}}$$

$$\overrightarrow{OL} = 2\vec{\imath} - 2\vec{\jmath} + 1\vec{k} - the\ axis\ of\ rotation \rightarrow \vec{u} = \frac{\overrightarrow{OL}}{|\overrightarrow{OL}|} - unit\ vector$$

$$|\overrightarrow{OL}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

Then,

$$\vec{u} = \frac{\overrightarrow{OL}}{|\overrightarrow{OL}|} = \frac{2\vec{\imath} - 2\vec{\jmath} + 1\vec{k}}{3} = \left(\frac{2}{3}\right)\vec{\imath} - \left(\frac{2}{3}\right)\vec{\jmath} + \left(\frac{1}{3}\right)\vec{k}$$

Conclusion,

$$\vec{\omega} = \omega \cdot \vec{u} = 3 \cdot \left(\left(\frac{2}{3}\right)\vec{\imath} - \left(\frac{2}{3}\right)\vec{\jmath} + \left(\frac{1}{3}\right)\vec{k}\right) = 2\vec{\imath} - 2\vec{\jmath} + 1\vec{k}$$

$$\boxed{\vec{\omega} = 2\vec{\imath} - 2\vec{\jmath} + 1\vec{k} \leftrightarrow \begin{cases} \omega_x = 2 \\ \omega_y = -2 \\ \omega_z = 1 \end{cases}}$$

Then,

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ r_x & r_y & r_z \end{vmatrix} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -2 & 1 \\ 4 & 1 & 3 \end{vmatrix} =$$

$$= \vec{\imath} \cdot \begin{vmatrix} -2 & 1 \\ 1 & 3 \end{vmatrix} - \vec{\jmath} \cdot \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} + \vec{k} \cdot \begin{vmatrix} 2 & -2 \\ 4 & 1 \end{vmatrix} = \vec{\imath} \cdot (-2 \cdot 3 - 1 \cdot 1) - \vec{\jmath} \cdot (2 \cdot 3 - 1 \cdot 4) + \vec{k} \cdot \big(2 \cdot 1 - 4 \cdot (-2)\big) =$$

$$= \vec{\imath} \cdot (-6 - 2) - \vec{\jmath} \cdot (6 - 4) + \vec{k} \cdot (2 + 8) = -8\vec{\imath} + 2\vec{\jmath} + 10\vec{k}$$

$$\boxed{\vec{v} = -8\vec{\imath} + 2\vec{\jmath} + 10\vec{k}}$$

$$v = |\vec{v}| = \sqrt{(-8)^2 + (2)^2 + (10)^2} = \sqrt{64 + 4 + 100} = \sqrt{168} = \sqrt{4 \cdot 42} = 2\sqrt{42} \approx 12.961481$$

$$\boxed{v = 2\sqrt{42} \approx 12.961481}$$

ANSWER:

$$\begin{cases} \vec{v} = -8\vec{\imath} + 2\vec{\jmath} + 10\vec{k} \\ v = 2\sqrt{42} \approx 12.961481 \end{cases}$$