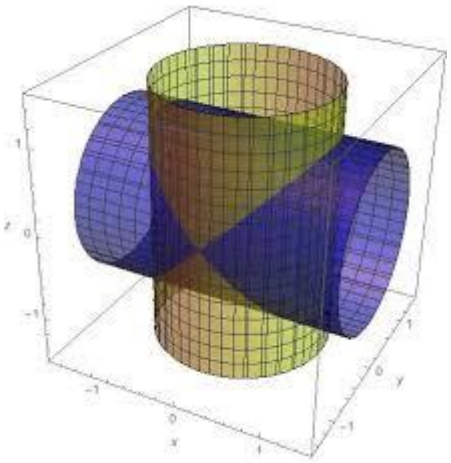
Answer on Question #77820 – Math – Geometry

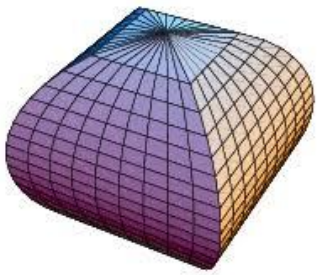
Question

Hi my geometry question is- A new part needs to be designed for a machine. The cube center of the part has a side length of 2 in. Each cylinder off the sides of the cube has a diameter and height that matches the sides of the cube with a 1 inch hole drilled out 1 inch deep. There are 4 cylinders. How much metal is needed to make one part?

Solution

As each of four cylinders is 1 in deep (the height of each cylinder is 1 in) we can say that every two cylinders form one cylinder if height 2 in, i.e. the same value that the side of a cube is. A solid formed by intersection of these two cylinders is the one that is drilled off the initial cube. The first hole is centered along the x axis, the second hole is centered along the y axis. The solid common to two (or three) right circular cylinders of equal radii intersecting at right angles is called the Steinmetz solid. Two cylinders intersecting at right angles are called a bicylinder.





The volume of the initial cube is:

$$V_{cube} = a^3 = 2^3 \text{ in}^3 = 8 \text{ in}^3$$

To find common volume of two intersecting cylinders we should subtract volume of bicylinder from the volume of the two cylinders added together.

The volume of bicylider is:

$$V_{bicyl} = \frac{16}{3} r^3$$

where r is the radius of a cylinder.

As diameter of a cylinder is 1 in then radius is

r=d/2=1/2 (in).

$$V_{bicyl} = \frac{16}{3} \times \left(\frac{1}{2}\right)^3 = \frac{16}{3 \times 8} = \frac{2}{3} (in^3)$$

The volume of one cylinder is:

$$V_{cyl} = \pi r^2 h$$

$$V_{cyl} = \pi \times \left(\frac{1}{2}\right)^2 \times 2 = \pi \times \frac{1}{2} \ (in^3)$$

The volume of the two cylinders added together is:

$$2V_{cyl} = 2 \times \pi \times \frac{1}{2} = \pi \ (in^3)$$

The volume of a solid obtained is:

$$V = V_{cube} - (2V_{cyl} - V_{bicyl}) = 8 - \left(\pi - \frac{2}{3}\right) = 8 + \frac{2}{3} - \pi = \frac{26}{3} - \pi \cong 5.53 \ (in^3)$$

The volume of a metal required is 5.53 in^3.

Answer: 5.53 in^3.