ANSWER on Question #77815 – Math – Differential Equations

QUESTION

$$(D^2 + 3DD' + 2D')z = \cos(x + 3y)$$

SOLUTION

As we know

$$F(D, D')z = f(x, y) \to z(x, y) = C.F. + P.I.$$

where

$$C.F.: F(D, D')z = 0$$

1 STEP: Let find C.F.

$$(D^2 + 3DD' + 2D')z = 0$$

Since $(D^2 + 3DD' + 2D')$ cannot be resolved into linear factors in D and D', hence

$$C.F. = Ae^{hx+ky},$$

where A, h, k are arbitrary constants.

$$D^2(Ae^{kx+ky}) = \frac{\partial^2}{\partial x^2}(Ae^{hx+ky}) = h^2 \cdot Ae^{hx+ky}$$

$$3DD'(Ae^{hx+ky}) = 3 \cdot \frac{\partial^2}{\partial x \partial y}(Ae^{hx+ky}) = 3hk \cdot Ae^{hx+ky}$$

$$2D'(Ae^{hx+ky}) = 2 \cdot \frac{\partial}{\partial y}(Ae^{hx+ky}) = 2k \cdot Ae^{hx+ky}$$

Then,

$$(D^2 + 3DD' + 2D')z = 0 \to A(h^2 + 3hk + 2k)e^{hx+ky} = 0 \to h^2 + 3hk + 2k = 0 \to$$

$$k(2 + 3h) = -h^2 \to \boxed{k = \frac{-h^2}{2 + 3h}}$$

Conclusion,

$$\boxed{\begin{cases} C.F. = \sum_{h=-\infty}^{+\infty} Ae^{hx+ky} \\ k = \dfrac{-h^2}{2+3h} \end{cases}}$$

2 STEP: Let find P.I.

$$P.I. = \frac{1}{D^2 + 3DD' + 2D'}\cos(x+3y) = \frac{1}{D^2 + 3DD' + 2D'}\cos(1\cdot x + 3\cdot y) =$$

$$= \frac{1}{(1\cdot i)^2 + 3\cdot(1\cdot i)\cdot(3\cdot i) + 2D'}\cos(1\cdot x + 3\cdot y) = \frac{1}{i^2 + 3\cdot 3i^2 + 2D'}\cos(x+3y) =$$

$$= \frac{1}{-1 + 3\cdot(-3) + 2D'}\cos(x+3y) = \frac{1}{-1 - 9 + 2D'}\cos(x+3y) = \frac{1}{2D' - 10}\cos(x+3y) =$$

$$= \frac{1}{2(D'-5)}\cos(x+3y) = \frac{(D'+5)}{2(D'-5)(D'+5)}\cos(x+3y) = \frac{1}{2}\cdot\frac{D'+5}{D'^2 - 25}\cos(x+3y) =$$

$$= \frac{1}{2}\cdot\frac{D'+5}{(3i)^2 - 25}\cos(x+3y) = \frac{1}{2}\cdot\frac{D'+5}{-9-25}\cos(x+3y) = \frac{1}{2}\cdot\frac{D'+5}{-34}\cos(x+3y) =$$

$$= -\frac{1}{68}\left(\frac{\partial}{\partial y} + 5\right)\cos(x+3y) = -\frac{1}{68}\left(\frac{\partial}{\partial y}(\cos(x+3y)) + 5\cdot\cos(x+3y)\right) =$$

$$= -\frac{1}{68}(-3\sin(x+3y) + 5\cos(x+3y))$$

Conclusion,

$$\boxed{P.I. = \frac{3}{68}\cdot\sin(x+3y) - \frac{5}{68}\cdot\cos(x+3y)}$$

Then,

$$z(x,y) = C.F. + P.I. = \sum_{h=-\infty}^{+\infty} Ae^{hx+ky} + \frac{3}{68}\cdot\sin(x+3y) - \frac{5}{68}\cdot\cos(x+3y)$$

$$\begin{cases} z(x,y) = \sum_{h=-\infty}^{+\infty} Ae^{hx+ky} + \frac{3}{68} \cdot \sin(x+3y) - \frac{5}{68} \cdot \cos(x+3y) \\ k = \frac{-h^2}{2+3h} \end{cases}$$

ANSWER:

$$\begin{cases} z(x,y) = \sum_{h=-\infty}^{+\infty} Ae^{hx+ky} + \frac{3}{68} \cdot \sin(x+3y) - \frac{5}{68} \cdot \cos(x+3y) \\ k = \frac{-h^2}{2+3h} \end{cases}$$