Question

The Earth is approximately 92.9 million miles from the Sun, and 240,000 miles from the Moon.
When the angle Sun-Earth-Moon is 25°, how far is the Moon from the Sun?

Solution
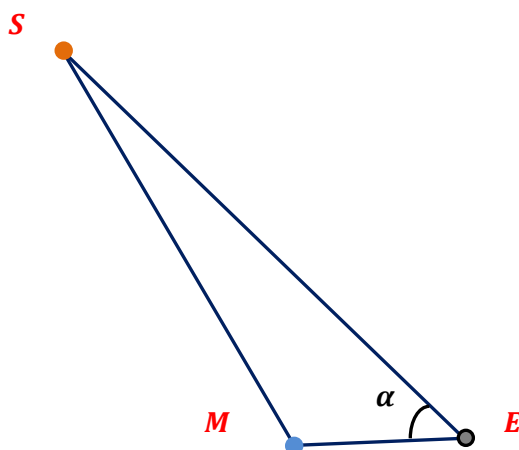
Let's set the Earth's position as point E, the Sun's position as point S and the Moon's position as point M. Then we have triangle SEM as illustrated in the Figure, where

$|SE| = 9.29 \cdot 10^7 \ miles$;

$|ME| = 2.4 \cdot 10^5 \ miles$;

$\alpha = 25^0$.

According to the law of cosines:

$$|MS|^2 = |SE|^2 + |ME|^2 - 2|SE||ME|cos\alpha,$$

$$|MS|^2 = (9.29 \cdot 10^7)^2 + (2.4 \cdot 10^5)^2 - 2 \cdot 9.29 \cdot 10^7 \cdot 2.4 \cdot 10^5 \cdot cos25^0 \ .$$

Concerning the fact that $cos25^0 = 0.9063$, we have:

$$|MS|^2 = (9.29 \cdot 10^7)^2 + (2.4 \cdot 10^5)^2 - 2 \cdot 9.29 \cdot 10^7 \cdot 2.4 \cdot 10^5 \cdot 0.9063$$

$$|MS|^2 = 86.3 \cdot 10^{14} + 5.76 \cdot 10^{10} - 44.59 \cdot 10^{12}$$

$$|MS|^2 = 863000 \cdot 10^{10} + 5.76 \cdot 10^{10} - 4459 \cdot 10^{10}$$

$$|MS|^2 = 858546.76 \cdot 10^{10}$$

$$|MS| = 926.6 \cdot 10^5 = 9.27 \cdot 10^7 \ miles.$$

So the Moon is 92.7 million miles from the Sun.