

Answer on Question #77576, Math / Differential Equations  
Consider  $z(x, y) = \ln(x^2 + y^2)$  :

$$\frac{\partial z}{\partial x} = \frac{1}{x^2 + y^2} \cdot (2x)$$
$$\frac{\partial^2 z}{\partial x^2} = 2 \cdot \frac{x^2 + y^2 - 2x^2}{(x^2 + y^2)^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2}$$

According to a symmetry of  $z(x, y)$ :

$$\frac{\partial^2 z}{\partial y^2} = \frac{2(x^2 - y^2)}{(x^2 + y^2)^2}$$

Then,

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2(y^2 - x^2)}{(x^2 + y^2)^2} + \frac{2(x^2 - y^2)}{(x^2 + y^2)^2} = 0$$