## Question #77546, Math / Geometry

By graphing the boundaries of a 2d figure on a graph using these inequalities  $\{3x - 2y > -12, y < 3, y > -3, x < 0, \text{ what is the volume and surface area of the 3d shape produced by revolving this 2d shape around the y-axis?$ 

## SOLUTION

A figure bounded by inequalities is shaded on the picture. After revolving this shape around the y-axis we obtain the frustum of a cone (see picture).



To determine the volume we calculate volumes of two cones (big with height 9 and radius 6 and small with height 3 and radius 3) and their subtraction is the required volume:

$$V_1 = \frac{1}{3}\pi \cdot 6^2 \cdot 9 = 108\pi;$$
  

$$V_2 = \frac{1}{3}\pi \cdot 3^2 \cdot 3 = 9\pi;$$
  

$$V = V_1 - V_2 = 108\pi - 9\pi = 99\pi.$$

To determine the surface area we calculate area of two cones (big with height 9 and small with height 3) and their subtraction is the required surface area:

 $\begin{aligned} A_1 &= \pi \cdot 6 \cdot \sqrt{6^2 + 9^2} = 6\sqrt{117}\pi = 18\sqrt{13}\pi; \\ A_2 &= \pi \cdot 3 \cdot \sqrt{3^2 + 3^2} = 3\sqrt{18}\pi = 6\sqrt{2}\pi; \\ A &= A_1 - A_2 = 18\sqrt{13}\pi - 6\sqrt{2}\pi = 6(3\sqrt{13} - \sqrt{2})\pi. \end{aligned}$