## Question \#77546, Math / Geometry

By graphing the boundaries of a 2d figure on a graph using these inequalities $\{3 x-2 y>$ $-12, y<3, y>-3, x<0$, what is the volume and surface area of the 3 d shape produced by revolving this 2 d shape around the $y$-axis?

## SOLUTION

A figure bounded by inequalities is shaded on the picture. After revolving this shape around the $y$-axis we obtain the frustum of a cone (see picture).


To determine the volume we calculate volumes of two cones (big with height 9 and radius 6 and small with height 3 and radius 3 ) and their subtraction is the required volume:

$$
\begin{aligned}
& V_{1}=\frac{1}{3} \pi \cdot 6^{2} \cdot 9=108 \pi \\
& V_{2}=\frac{1}{3} \pi \cdot 3^{2} \cdot 3=9 \pi \\
& V=V_{1}-V_{2}=108 \pi-9 \pi=99 \pi .
\end{aligned}
$$

To determine the surface area we calculate area of two cones (big with height 9 and small with height 3) and their subtraction is the required surface area:
$A_{1}=\pi \cdot 6 \cdot \sqrt{6^{2}+9^{2}}=6 \sqrt{117} \pi=18 \sqrt{13} \pi$;
$A_{2}=\pi \cdot 3 \cdot \sqrt{3^{2}+3^{2}}=3 \sqrt{18} \pi=6 \sqrt{2} \pi$;
$A=A_{1}-A_{2}=18 \sqrt{13} \pi-6 \sqrt{2} \pi=6(3 \sqrt{13}-\sqrt{2}) \pi$.

