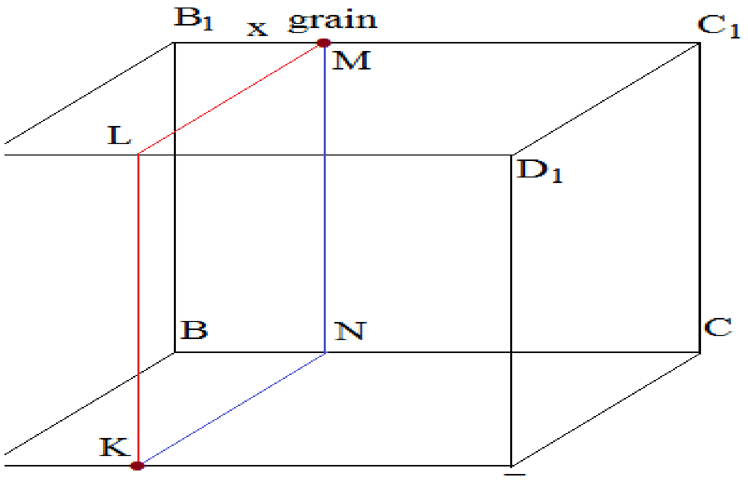
Answer on Question #77515 – Math – Other

An ant lives on the surface of a cube with edges of length 7 cm. It is currently located on an edge x cm from one of its ends. While traveling on the surface of the cube, it has to reach the grain located on the opposite edge (also at a distance x cm from one of its ends).

(i) What is the length of the shortest route to the grain if $x = 2$ cm? How many routes of this length are there?

(ii) Find an x for which there are four distinct shortest length routes to the grain.

Solution



(i) If the ant can travel on the surface of the cube, there are two distinct shortest length routes to the grain $(0 < x < 7)$

$$KL = LM = KN = NM = 7\ cm$$

$$d = 7\ cm + 7\ cm = 14\ cm$$

(ii) If the ant can travel on the surface of the cube, there are two distinct shortest length routes to the grain for all $x \in (0,7)$.

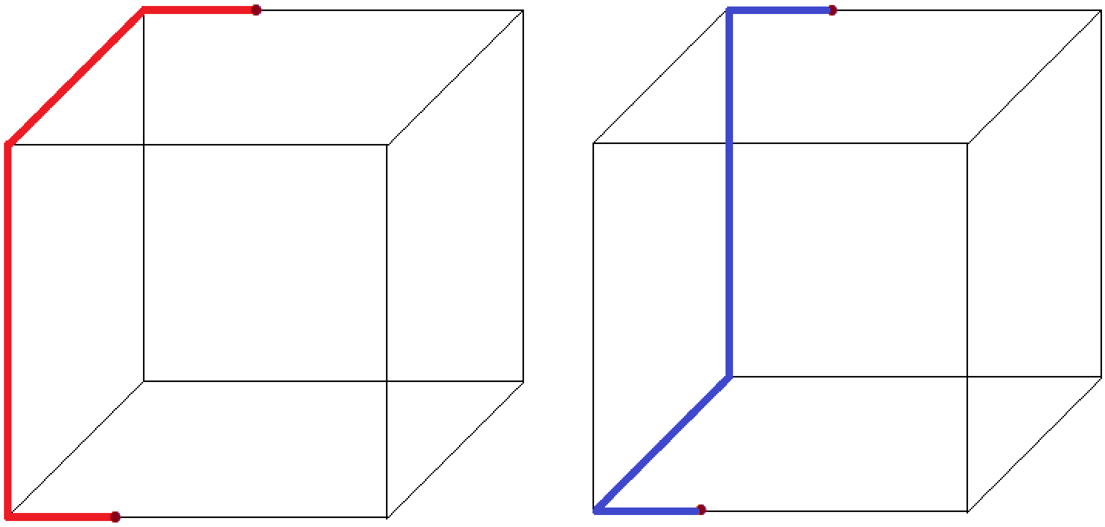
The value of x for which there are four distinct shortest length routes to the grain does not exist.

If the ant can travel only on the edges of the cube, there are two distinct shortest length routes to the grain for all $x \in (0, 3.5) \cup (3.5, 7)$

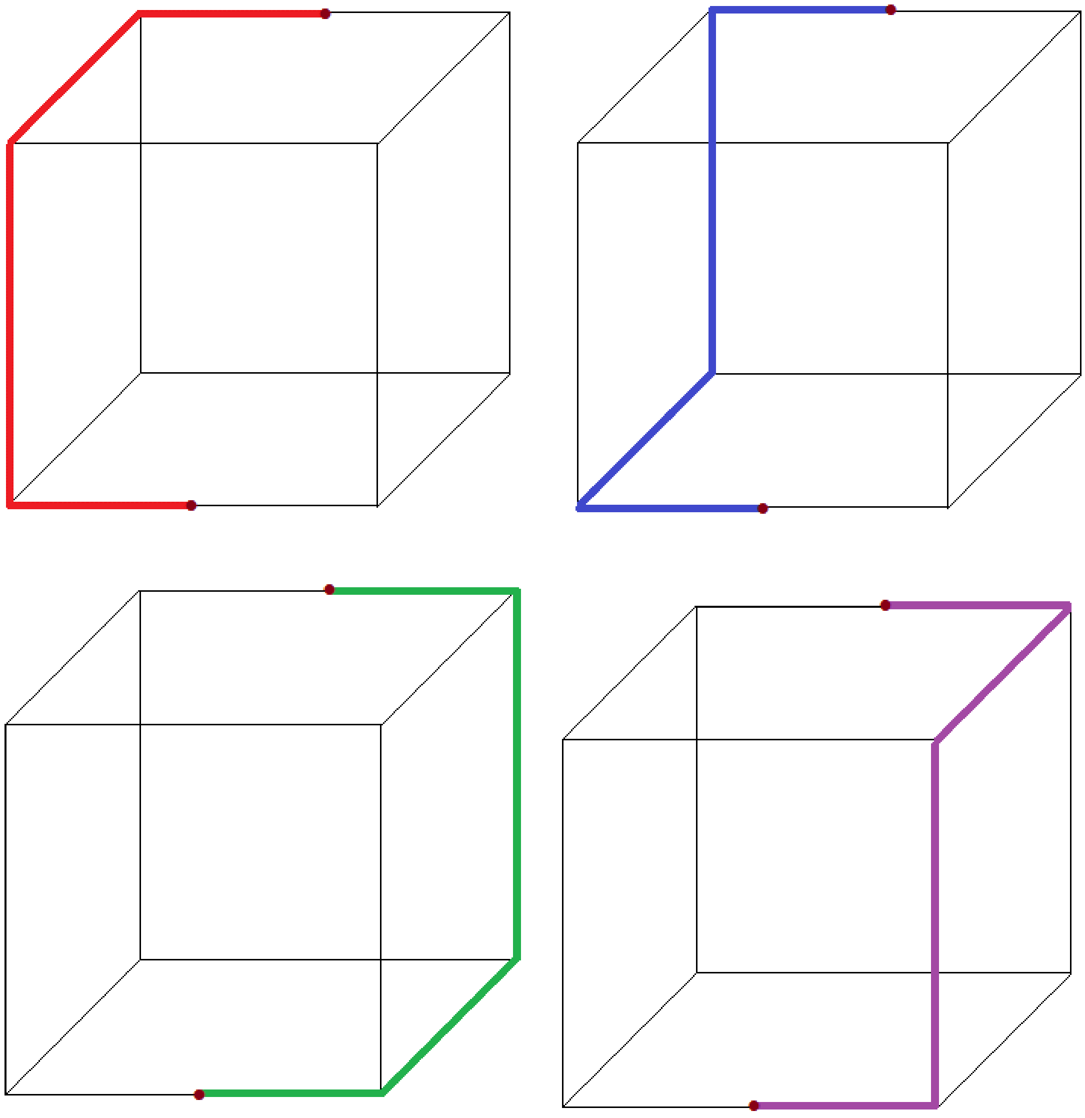
$$d = x\ cm + 7\ cm + 7\ cm + x\ cm = 14\ cm + 2x\ cm$$

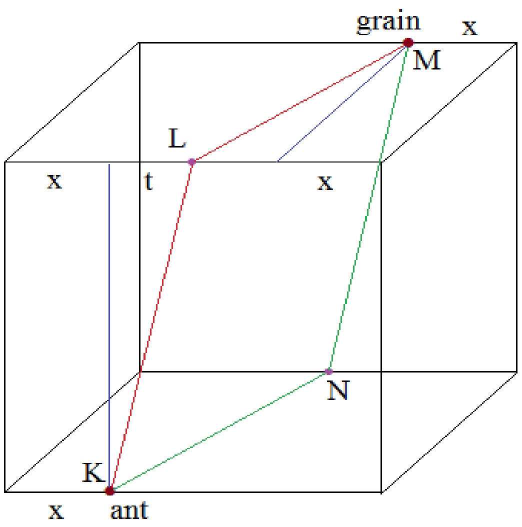
When $x\ cm = 2\ cm$

$$d = 18\ cm$$



The value of x, for which there are four distinct shortest length routes to the grain
$x = 3.5\ cm, d = 3.5\ cm + 7\ cm + 7\ cm + 3.5\ cm = 21\ cm$





(i) If the ant can travel on the surface of the cube, there are two distinct shortest length routes to the grain $(0 < x \leq 3.5)$

$$KL = \sqrt{7^2 + t^2}, LM = \sqrt{7^2 + (7 - 2x - t)^2}$$

$$d = \sqrt{7^2 + t^2} + \sqrt{7^2 + (7 - 2x - t)^2}$$

Find the first derivative with respect to t

$$\frac{d}{dt}(d) = \frac{2t}{2\sqrt{7^2 + t^2}} - \frac{2(7 - 2x - t)}{2\sqrt{7^2 + (7 - 2x - t)^2}}$$

Find the critical number(s)

$$\frac{d}{dt}(d) = 0 => \frac{t}{\sqrt{49 + t^2}} - \frac{(7 - 2x - t)}{\sqrt{49 + (7 - 2x - t)^2}} = 0$$

$$t^2(49 + (7 - 2x - t)^2) = (49 + t^2)(7 - 2x - t)^2$$

$$49t^2 + t^2(7 - 2x - t)^2 = 49(7 - 2x - t)^2 + t^2(7 - 2x - t)^2$$

$$t^2 = (7 - 2x - t)^2$$

$$t = 7 - 2x - t$$

$$t = 3.5 - x$$

$If\ 0 < t < 3.5 - x, \frac{d}{dt}(d) < 0, d\ decreases$

$If\ 3.5 - x < t < 3.5, \frac{d}{dt}(d) > 0, d\ increases$

The distance has the minimum at $t = 3.5 - x$

$$d = 2\sqrt{49 + (3.5 - x)^2}$$

When $x = 2\ cm$

$$d = 2\sqrt{49 + (3.5 - 2)^2} = \sqrt{205}$$

$$d = \sqrt{205}\ cm$$

If the ant can travel on the surface of the cube, there are two distinct shortest length routes to the grain for all $x \in (0, 3.5]$.

(ii) The value of x for which there are four distinct shortest length routes to the grain does not exist.

Answer provided by https://www.AssignmentExpert.com