

Answer on Question #77468 – Math – Algebra

Question

The equality $\cos x \cdot (\tan x + \sin x \cdot \cot x) = \sin x + (\cos x)^2$ is true for all $x \in R: x \neq \frac{\pi}{2} + \frac{\pi k}{2}, k \in Z$.

Solution

The function $y = \tan x$ exists for all $x \in R: x \neq \frac{\pi}{2} + \pi k, k \in Z$.

The function $y = \cot x$ exists for all $x \in R: x \neq \pi k, k \in Z$.

So consider the expression $\cos x \cdot (\tan x + \sin x \cdot \cot x)$ for all

$x \in R: x \neq \frac{\pi}{2} + \frac{\pi k}{2}, k \in Z$.

$$\begin{aligned}\cos x \cdot (\tan x + \sin x \cdot \cot x) &= \cos x \cdot \left(\frac{\sin x}{\cos x} + \sin x \cdot \frac{\cos x}{\sin x} \right) = \cos x \cdot \left(\frac{\sin x}{\cos x} + \cos x \right) \\ &= \sin x + (\cos x)^2 \blacksquare\end{aligned}$$