

Answer on Question #77428 – Math – Differential Equations

Question

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 1 + 3x + x^2.$$

Solution

The corresponding homogeneous equation is $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$. So, characteristic equation will be

$$r^2 + 3r + 2 = (r + 1)(r + 2) = 0.$$

Then the complementary solution will be

$$y_c = C_1e^{-x} + C_2e^{-2x}.$$

The nonhomogeneous equation has $f(x) = 1 + 3x + x^2$. We will search a particular solution in the next general quadratic polynomial form:

$$y_p = A + Bx + Cx^2.$$

Then $\frac{dy}{dx} = B + 2Cx$ and $\frac{d^2y}{dx^2} = 2C$. Substitute them into the equation:

$$2C + 3(B + 2Cx) + 2(A + Bx + Cx^2) = 1 + 3x + x^2.$$

The corresponding terms on both sides should have the same coefficients. Hence, we obtain:

$$2C = 1,$$

$$6C + 2B = 3,$$

$$2C + 3B + 2A = 1.$$

Now we have the next solution for unknown coefficients:

$$C = \frac{1}{2},$$

$$B = 0,$$

$$A = 0.$$

The general solution of equation is

$$y = y_c + y_p = C_1e^{-x} + C_2e^{-2x} + \frac{x^2}{2}.$$

Answer: $y = C_1e^{-x} + C_2e^{-2x} + \frac{x^2}{2}$.