Answer on Question #77403 – Math – Calculus

Question

1) Find the result of the following integrals.

a) ∫▒ 〖(2+y)^2 dy〗

b) ∫▒(x²+x+1)/√x dx

2) Find the results of the following integrals by using Substitution rules.

a) ∫_0^1▒ 〖xe〗 ^(-x²) dx

b) ∫_0^1▒ 〖(e^z+1)/(e^z+z) dz〗

c) ∫▒ 〖dx/(ax+b)(a≠0)〗

Solution

1. Find the result of the following integrals.

a)

$$\int (2+y)^2 dy = \int (4+4y+y^2)dy = 4y + 2y^2 + \frac{y^3}{3} + C.$$

b)

$$\int \frac{x^2+x+1}{\sqrt{x}}dx = \int \left(x^{3/2}+x^{1/2}+x^{-1/2}\right)dx = \frac{x^{3/2+1}}{\frac{3}{2}+1} + \frac{x^{1/2+1}}{\frac{1}{2}+1} + \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2\sqrt{x} + C.$$

2. Find the results of the following integrals by using Substitution rules.

a)

$$\int_0^1 xe^{-x^2}dx = \frac{1}{2}\int_0^1 e^{-x^2}d(x^2) = \begin{vmatrix} \text{subs. } x^2 = t; \\ x_1 = 0 \Rightarrow t_1 = 0; \\ x_2 = 1 \Rightarrow t_2 = 1. \end{vmatrix} = \frac{1}{2}\int_0^1 e^{-t}dt = \frac{1}{2}[-e^{-t}]_0^1$$

$$= \frac{1}{2}[-e^{-1}+e^0] = \frac{1}{2}\left(1-\frac{1}{e}\right).$$

b)

$$\int_0^1 \frac{e^z+1}{e^z+z}dz = \int_0^1 \frac{d(e^z+z)}{e^z+z} = \begin{vmatrix} \text{subs. } e^z+z = u; \\ z_1 = 0 \Rightarrow u_1 = 1; \\ z_2 = 1 \Rightarrow u_2 = 1+e. \end{vmatrix} = \int_1^{1+e}\frac{du}{u} = [\ln u]_1^{1+e}$$

$$= \ln(1+e) - \ln 1 = \ln(1+e).$$

c)

$$\int \frac{dx}{ax+b} = \frac{1}{a}\int \frac{d(ax+b)}{ax+b} = |\text{subs. } ax+b = w; | = \frac{1}{a}\int \frac{dw}{w} = \frac{1}{a}\ln w + C$$

$$= \frac{1}{a}\ln(ax+b) + C.$$