Answer on Question #77402 – Math – Calculus

Question

Using the definition of the derivative find the derivative of the function f(a)=(2a+1)/(a+3).

Solution

$$f(a)=\frac{2a+1}{a+3},\qquad a+3\neq 0$$

$$f'(a)=\lim_{\Delta a\to 0}\frac{\Delta\left(\frac{2a+1}{a+3}\right)}{\Delta a}=\lim_{\Delta a\to 0}\frac{\frac{2a+1+\Delta a}{a+3+\Delta a}-\frac{2a+1}{a+3}}{\Delta a}=$$

$$=\lim_{\Delta a\to 0}\frac{(2a+1+\Delta a)(a+3)-(2a+1)(a+3+\Delta a)}{\Delta a(a+3)(a+3+\Delta a)}=\frac{1}{(a+3)^2}*$$

$$*\lim_{\Delta a\to 0}\frac{\big(2a+1+\Delta(2a+1)\big)*\big((a+3)-\Delta(a+3)\big)*(2a+1)}{\Delta a}=$$

$$=\frac{1}{(a+3)^2}\lim_{\Delta a\to 0}\frac{(2a+1)(a+3)+(a+3)\Delta(2a+1)-(2a+1)(a+3)-(2a+1)\Delta(a+3)}{\Delta a}=$$

$$=\frac{1}{(a+3)^2}\lim_{\Delta a\to 0}\frac{(a+3)\Delta(2a+1)-(2a+1)\Delta(a+3)}{\Delta a}=$$

$$=\frac{1}{(a+3)^2}\left((a+3)*\lim_{\Delta a\to 0}\frac{\Delta(2a+1)}{\Delta a}-(2a+1)*\lim_{\Delta a\to 0}\frac{\Delta(a+3)}{\Delta a}\right)=\frac{1}{(a+3)^2}\big((a+3)*2-$$

$$-(2a+1)*1)=\frac{2a+6-2a-1}{(a+3)^2}=\frac{5}{(a+3)^2}$$

Answer: $\frac{5}{(a+3)^2}$.