Question

If there are 3 misprints in a book of 1000 pages, find the probability that a given page will contain

a) No misprint.
b) More than 2 misprints.

Solution

Assume that these misprints are randomly distributed throughout the book and x, the number of the misprints per page has a Poisson distribution.

Total number of pages $= 1000$.

$$P = \frac{3}{1000} = 0.003$$
$$n = 1$$
$$m = nP = 1 \cdot 0.003 = 0.003$$
$$P(r) = \frac{e^{-m} \cdot m^r}{r!}$$

a) $P(0) = \dfrac{e^{-0.003} \cdot 0.003^0}{0!} = e^{-0.003} \approx 0.997$

Answer: $P(0) = e^{-0.003} \approx 0.997$

b) $\mathrm{P}(r = 3) = \dfrac{e^{-0.003} \cdot 0.003^3}{3!} = 13.5 \cdot e^{-0.003} \times 10^{-9} \approx$
$\approx 4.45 \times 10^{-9} = 0.00000000445$

$$P(0) = \frac{e^{-0.003} \cdot 0.003^0}{0!} = e^{-0.003}$$
$$P(1) = \frac{e^{-0.003} \cdot 0.003^1}{1!} = e^{-0.003} \cdot 0.003$$
$$P(2) = \frac{e^{-0.003} \cdot 0.003^2}{2!} = e^{-0.003} \cdot 0.0000045$$
$$P(r > 2) = 1 - (P(0) + P(1) + P(2))$$
$$P(r > 2) = 1 - (e^{-0.003} + e^{-0.003} \cdot 0.003 + e^{-0.003} \cdot 0.0000045) =$$
$$= 1 - e^{-0.003} \cdot 1.0030045 \approx 4.49 \times 10^{-9}$$

Answer: $P(r > 2) = 1 - e^{-0.003} \cdot 1.0030045 \approx 4.49 \times 10^{-9}$
$$= 0.00000000449$$