Question #77340, Math, Differential Equations

$y'' + 3y' - 10y = 3x^2$

Solution The characteristic equation is $k^2 + 3k - 10 = 0$ and its solutions

$$k_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1} = \frac{-3 \pm 7}{2} = -5; 2.$$

The general solution of the equation $y'' + 3y' - 10y = 0$ is $y = C_1 e^{-5x} + C_2 e^{2x}$. The partial solution of the equation $y'' + 3y' - 10y = 3x^2$ we'll find as $y_0 = ax^2 + bx + c$, where a, b, c are unknown real numbers.

$y_0' = 2ax + b$,

$y_0'' = 2a$ and

$2a + 3(2ax + b) - 10(ax^2 + bx + c) = 3x^2$,

$-10ax^2 + (6a - 10b)x + (2a + 3b - 10c) = 3x^2 + ox + 0$, from where

$$\begin{cases} -10a = 3 \\ 6a - 10b = 0 \\ 2a + 3b - 10c = 0 \end{cases} \Rightarrow \begin{cases} a = -\frac{3}{10} = -0.3 \\ b = \frac{6}{10}a = -\frac{18}{100} = -0.18 \\ c = \frac{2a+3b}{10} = \frac{-0.6-0.54}{10} = -0.114 \end{cases}$$

and

$$y_0 = -0.3x^2 - 0.18x - 0.114.$$

The general solution of the equation $y'' + 3y' - 10y = 3x^2$ is the sum of the general solution of the equation $y'' + 3y' - 10y = 0$ and the partial solution of the equation $y'' + 3y' - 10y = 3x^2$, so

$$y = C_1 e^{-5x} + C_2 e^{2x} - 0.3x^2 - 0.18x - 0.114.$$