ANSWER on Question #77331 Math. Calculus

QUESTION

A 6 foot boy throws the javelin with an initial speed of 87 feet per second at an angle of 38° with the horizontal. What is the maximum height reached?

SOLUTION

The maximum height h_{max} consists of two parts:

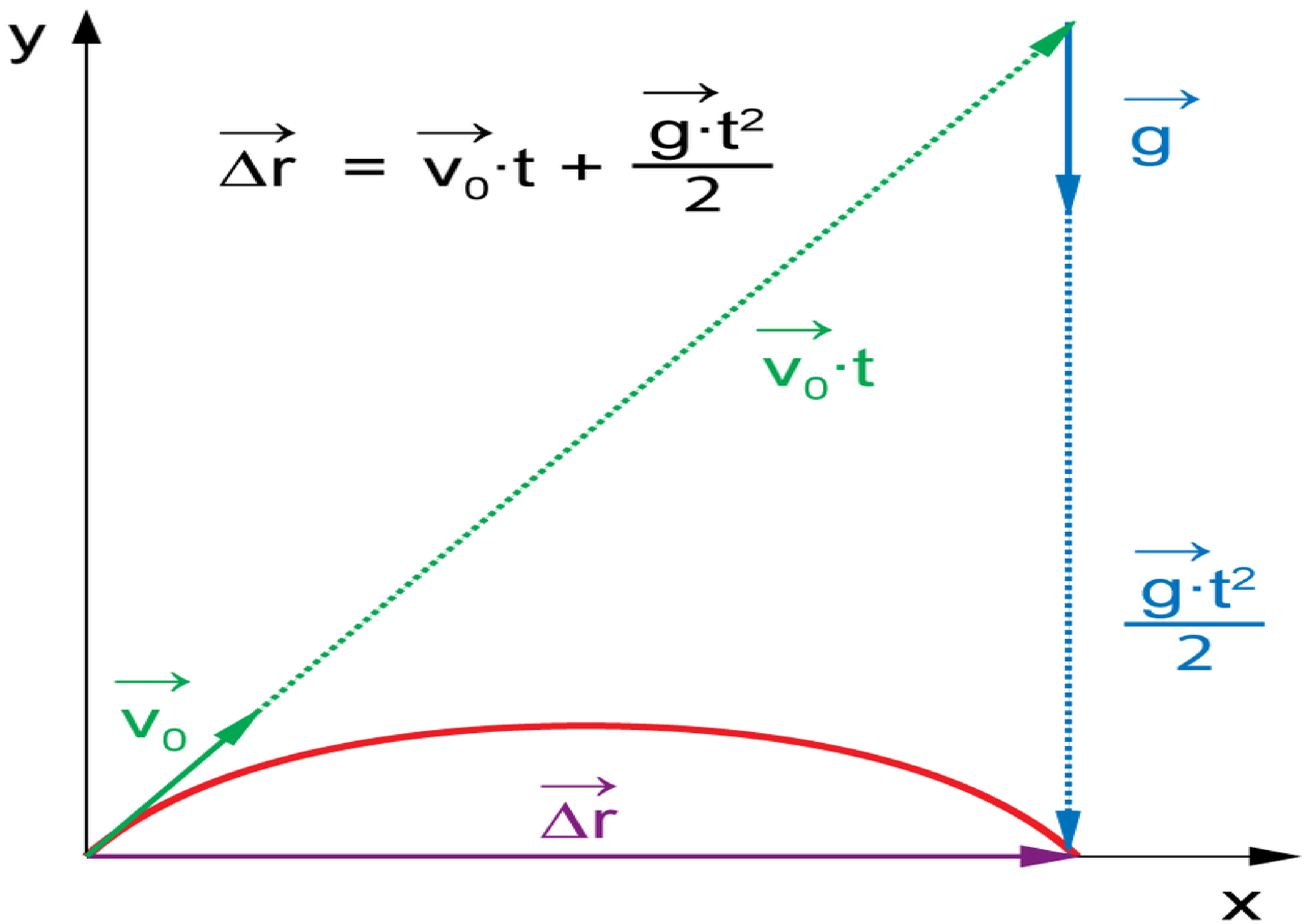
1) initial throw height - $h_1 = 6\ feet$

2) the height at which the spear would have risen had it been thrown from the Earth - $h_2 = ?$

Conclusion,

That is, our task is to find a formula that shows the maximum height if we know the initial speed v_0

and angle α of the throw.



The equation of motion has the form

$$\vec{r} = \overrightarrow{v_0}t + \frac{\vec{g}t^2}{2}$$

In the projections on the coordinate axes:

1) Rise

$$Oy : y = v_y t - \frac{gt^2}{2}$$

$$Ox : x = v_x t$$

And

$$\begin{cases} v_y = v_o \cdot \sin\alpha \\ v_x = v_0 \cdot \cos\alpha \end{cases}$$

2) Declivity

$$Oy : y = h_2 - \frac{gT^2}{2}$$

$$Ox : x = (v_0 \cdot \cos\alpha)T$$

We are interested in only the first part of the movement - the rise. The rise will occur as long as the force of gravity is not completely "stop" the body. In the formula form this condition has the form

$$\begin{cases} v_y = v_o \cdot \sin\alpha - gt \\ v_y = 0 \end{cases} \rightarrow v_o \cdot \sin\alpha - gt_{rise} = 0 \rightarrow t_{rise} = \frac{v_0 \cdot \sin\alpha}{g}$$

Conclusion,

$$\boxed{t_{rise} = \frac{v_0 \cdot \sin\alpha}{g}}$$

Then,

$$h_2 = y(t_{rise}) = (v_0 \cdot \sin\alpha) \cdot \left(\frac{v_0 \cdot \sin\alpha}{g}\right) - \frac{g}{2} \cdot \left(\frac{v_0 \cdot \sin\alpha}{g}\right)^2 = \frac{v_0^2 \cdot \sin^2\alpha}{g} - \frac{g \cdot v_0^2 \cdot \sin^2\alpha}{2g^2} =$$

$$= \frac{2 \cdot v_0^2 \cdot \sin^2\alpha}{2g} - \frac{v_0^2 \cdot \sin^2\alpha}{2g} = \frac{v_0^2 \cdot \sin^2\alpha}{2g}$$

Conclusion,

$$\boxed{h_2 = \frac{v_0^2 \cdot \sin^2 \alpha}{2g}}$$

Then,

$$h_{max} = h_1 + h_2 = 6 + \frac{v_0^2 \cdot \sin^2 \alpha}{2g} \approx 6 + \frac{(87)^2 \cdot \sin^2(38°)}{2 \cdot (32.174)} = 50.58\ (feet)$$

$$\boxed{h_{max} \approx 50.58\ feet}$$

ANSWER

$$h_{max} \approx 50.58\ feet$$