Answer on Question # 77329 – Math – Calculus

Question

Find the fourth roots of $-1 - i\sqrt{3}$

Solution

Trigonometric form of $z = -1 - i\sqrt{3}$:

$$|z| = \left|-1 - i\sqrt{3}\right| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$x < 0 \; and \; y < 0 \to \varphi = \pi + artan\left(\frac{|y|}{|x|}\right) = \pi + artan\left(\frac{\left|-\sqrt{3}\right|}{|-1|}\right) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$z = 2\left(\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right)$$

The n^{th} roots of z:

$$z_k = \sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi + 2\pi k}{n}\right) + isin\left(\frac{\varphi + 2\pi k}{n}\right)\right), where \; 0 \le k \le n-1$$

The 4^{th} roots of z:

$$k = 0:$$

$$z_0 = \sqrt[4]{|2|}\left(\cos\left(\frac{\frac{4\pi}{3} + 2\pi * 0}{4}\right) + isin\left(\frac{\frac{4\pi}{3} + 2\pi * 0}{4}\right)\right) = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right)$$

$$z_0 = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right) \; or \; z_0 = \sqrt[4]{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$k = 1:$$

$$z_1 = \sqrt[4]{|2|}\left(\cos\left(\frac{\frac{4\pi}{3} + 2\pi * 1}{4}\right) + isin\left(\frac{\frac{4\pi}{3} + 2\pi * 1}{4}\right)\right) = \sqrt[4]{2}\left(\cos\left(\frac{5\pi}{6}\right) + isin\left(\frac{5\pi}{6}\right)\right)$$

$$z_1 = \sqrt[4]{2}\left(\cos\left(\frac{5\pi}{6}\right) + isin\left(\frac{5\pi}{6}\right)\right) \; or \; z_1 = \sqrt[4]{2}\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$k = 2:$$

$$z_2 = \sqrt[4]{|2|}\left(\cos\left(\frac{\frac{4\pi}{3} + 2\pi * 2}{4}\right) + isin\left(\frac{\frac{4\pi}{3} + 2\pi * 2}{4}\right)\right) = \sqrt[4]{2}\left(\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right)$$

$$z_2 = \sqrt[4]{2}\left(\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right)\ or\ z_2 = \sqrt[4]{2}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$k = 3:$$

$$z_3 = \sqrt[4]{|2|}\left(\cos\left(\frac{\frac{4\pi}{3} + 2\pi * 3}{4}\right) + isin\left(\frac{\frac{4\pi}{3} + 2\pi * 3}{4}\right)\right) = \sqrt[4]{2}\left(\cos\left(\frac{11\pi}{6}\right) + isin\left(\frac{11\pi}{6}\right)\right)$$

$$z_3 = \sqrt[4]{2}\left(\cos\left(\frac{11\pi}{6}\right) + isin\left(\frac{11\pi}{6}\right)\right)\ or\ z_3 = \sqrt[4]{2}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$$

Answer:

$$z_0 = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{3}\right) + isin\left(\frac{\pi}{3}\right)\right)\ or\ z_0 = \sqrt[4]{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$z_1 = \sqrt[4]{2}\left(\cos\left(\frac{5\pi}{6}\right) + isin\left(\frac{5\pi}{6}\right)\right)\ or\ z_1 = \sqrt[4]{2}\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$z_2 = \sqrt[4]{2}\left(\cos\left(\frac{4\pi}{3}\right) + isin\left(\frac{4\pi}{3}\right)\right)\ or\ z_2 = \sqrt[4]{2}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$z_3 = \sqrt[4]{2}\left(\cos\left(\frac{11\pi}{6}\right) + isin\left(\frac{11\pi}{6}\right)\right)\ or\ z_3 = \sqrt[4]{2}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$$

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