Question #77328, Math / Calculus

z_1=1-i√3 and z_2=-√2+i√2
Find (z_2)^6

Answer

$$z_2 = -\sqrt{2} + i\sqrt{2}.$$

We use de Moivre's formula

$$(\cos\phi + i\sin\phi)^n = \cos(n\phi) + i\sin(n\phi).$$

Now we have to make the trigonometric representation of complex number z_2.

$$z_2 = |z_2| \cdot (\cos\phi + i\sin\phi),$$

where

$$|z_2| = \sqrt{a^2 + b^2} = \sqrt{\left(-\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{2+2} = 2;$$

$$\phi = \arctan\left(\frac{b}{a}\right) + \pi = \arctan\frac{\sqrt{2}}{-\sqrt{2}} + \pi = \arctan(-1) + \pi = \frac{3\pi}{4} + \pi = \frac{7\pi}{4}.$$

So we have

$$z_2 = 2 \cdot \left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right);$$

$$(z_2)^6 = 2^6 \cdot \left(\cos 6 \cdot \frac{7\pi}{4} + i\sin 6 \cdot \frac{7\pi}{4}\right) = 64 \cdot \left(\cos\frac{21\pi}{2} + i\sin\frac{21\pi}{2}\right)$$
$$= 64 \cdot \left(\cos\left(10\pi + \frac{\pi}{2}\right) + i\sin\left(10\pi + \frac{\pi}{2}\right)\right) = 64 \cdot \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 64 \cdot (0 + i) = 64i.$$

Answer: $64i$.