Answer on Question #77326 – Math – Calculus

Question

$z_1 = 1 - i\sqrt{3}$ and $z_2 = -\sqrt{2} + i\sqrt{2}$. Find $z_1 \times z_2$ in trigonometric form.

Solution

Trigonometric form of a complex number:

$$z = |z| * \left(\cos(\varphi) + isin(\varphi)\right)$$

$$|z| = \sqrt{x^2 + y^2}, \text{ where } z = x + iy$$

$$\varphi = \arg(z) = \begin{cases} arctan\left(\frac{y}{x}\right), & if\ x > 0\ and\ y \geq 0 \\ \pi - arctan\left(\frac{|y|}{|x|}\right), & if\ x < 0\ and\ y \geq 0 \\ \pi + arctan\left(\frac{|y|}{|x|}\right), & if\ x < 0\ and\ y < 0 \\ 2\pi - arctan\left(\frac{|y|}{|x|}\right), & if\ x > 0\ and\ y < 0 \\ \frac{\pi}{2}, & if\ x = 0\ and\ y > 0 \\ \frac{3\pi}{2}, & if\ x = 0\ and\ y < 0 \end{cases}$$

Trigonometric form of z_1:

$$|z_1| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$x > 0\ and\ y < 0 \rightarrow \varphi_1 = 2\pi - artan\left(\frac{|-\sqrt{3}|}{1}\right) = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$z_1 = 2\left(\cos\left(\frac{5\pi}{3}\right) + isin\left(\frac{5\pi}{3}\right)\right)$$

Trigonometric form of z_2:

$$|z_2| = \sqrt{\left(-\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{2+2} = \sqrt{4} = 2$$

$$x < 0\ and\ y \geq 0 \rightarrow\ \varphi_2 = \pi - \arctan\left(\frac{\sqrt{2}}{|-\sqrt{2}|}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$z_2 = 2\left(\cos\left(\frac{3\pi}{4}\right) + isin\left(\frac{3\pi}{4}\right)\right)$$

Multiplication of complex numbers in trigonometric form:

$$z_1z_2 = |z_1| * |z_2|\left(cos(\varphi_1 + \varphi_2) + isin(\varphi_1 + \varphi_2)\right)$$

$$z_1z_2 = 2 * 2\left(\cos\left(\frac{5\pi}{3} + \frac{3\pi}{4}\right) + isin\left(\frac{5\pi}{3} + \frac{3\pi}{4}\right)\right) = 4\left(\cos\left(\frac{29\pi}{12}\right) + isin\left(\frac{29\pi}{12}\right)\right)$$

Answer: $z_1z_2 = 4\left(\cos\left(\frac{29\pi}{12}\right) + isin\left(\frac{29\pi}{12}\right)\right)$.