Answer on Question #77264 – Math – Differential Geometry | Topology

Calculate the first fundamental forms of the following surfaces:

Question

1) Sphere: σ(θ,φ) =(cosθcosφ, cosθsinφ,sinθ)

Solution

$\sigma(\theta,\varphi) = (cos\theta cos\varphi, cos\theta sin\varphi, sin\theta)$

$\sigma_\theta(\theta,\varphi) = (-sin\theta cos\varphi, -sin\theta sin\varphi, cos\theta)$

$\sigma_\varphi(\theta,\varphi) = (-cos\theta sin\varphi, cos\theta cos\varphi, 0)$

$E = \sigma_\theta(\theta,\varphi)\cdot\sigma_\theta(\theta,\varphi) = sin^2\theta cos^2\varphi + sin^2\theta sin^2\varphi + cos^2\theta = 1$

$F = \sigma_\theta(\theta,\varphi)\cdot\sigma_\varphi(\theta,\varphi) = sin\theta cos\theta sin\varphi cos\varphi - sin\theta cos\theta sin\varphi cos\varphi = 0$

$G = \sigma_\varphi(\theta,\varphi)\cdot\sigma_\varphi(\theta,\varphi) = cos^2\theta sin^2\varphi + cos^2\theta cos^2\varphi = cos^2\theta$

Ans.: $d\sigma^2 = d\theta^2 + cos^2\theta d\varphi^2$

Question

2) A generalized cylinder: σ(u,v) =γ(u)+Va

Solution

$\sigma_u = y'(u), \sigma_v = a$

$\sigma_u^2 = 1, \sigma_v^2 = a^2, \sigma_u\sigma_v = ay'(u)$

Ans.: $d\sigma^2 = du^2 + 2ay'(u)dudv + a^2dv^2$