Question #77239

1 If f is function from A to B and g is function B to C and both f and g are onto. Show that $g \circ f$ is also onto. Is $g \circ f$ one-to-one if both f and g are one-to-one.

Solution

Let choose an arbitrary $c \in C$, since $g : B \to C$ is onto map, then there exists $b \in B$ such that $g(b) = c$. Since $f : A \to B$ is onto map, there exists $a \in A$ such that $f(a) = b$. Therefore, $c = g(b) = g(f(a))$ and it shows that $g \circ f$ is onto map.

For every $a_1 \neq a_2$ from one-to-one property of f we have $f(a_1) \neq f(a_2)$ and from one-to-one property of g we have $g(f(a_1)) \neq g(f(a_2))$. So, if both f and g are one-to-one maps the map $g \circ f$ is also one-to-one.

2 Let f, g and $h : \mathbb{R} \to \mathbb{R}$ be defined by ($\mathbb{R}$ is the set of real numbers) $f(x) = x+2$, $g(x) = (1+x^2)^{-1}$, $h(x) = 3$. Compute $f^{-1}g(x)$ and $hf(gf^{-1})(hf(x))$.

Solution

If $f(x) = x + 2$, then $f^{-1} = x - 2$ and $f^{-1}g(x) = f^{-1}((1 + x^2)^{-1}) = (1 + x^2)^{-1} - 2$.

$h(f(x)) = h(x + 2) = 3$,

$g(f^{-1})(3) = g(3 - 2) = g(1) = 2^{-1} = 0.5$,

$hf(0.5) = h(0.5 + 2) = h(2.5) = 3$, so

$hf(gf^{-1})(hf(x)) = 3$.